Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

A large bag contains counters of different colours.
Find the number of arrangements for the following selections

a 5 counters all of different colours,

b 5 counters where 3 are red and 2 are blue,

c 7 counters where 2 are red and 5 are green,

d 10 counters where 4 are blue and 6 are yellow,

e 20 counters where 2 are yellow and 18 are black.

Solution:

a $5! = 120$

b $\frac{5!}{3!2!} = 10$

c $\frac{7!}{5!2!} = \frac{7 \times 6}{2} = 21$

d $\frac{10!}{4!6!} = \frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 210$

e $\frac{20!}{18!2!} = \frac{20 \times 19}{2} = 190$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

A bag contains 4 red, 3 green and 8 yellow beads.
Five beads are selected at random from the bag without replacement.
Find the probability that they are

a 5 yellow beads,

b 2 red and 3 yellow,

c 4 red and 1 green.

Solution:

a

$$\frac{8}{15}\times\frac{7}{14}\times\frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}$$

$$= \frac{8}{429} \text{ or } 0.0186$$

b

$$\frac{4}{15}\times\frac{3}{14}\times\frac{8}{13}\times\frac{7}{12}\times\frac{6}{11}\times\frac{5!}{2!3!}$$

$$= \frac{16}{143} \quad \text{or} \quad 0.112$$

c

$$\frac{4}{15}\times\frac{3}{14}\times\frac{2}{13}\times\frac{1}{12}\times\frac{3}{11}\times\frac{5!}{4!1!}$$

$$= 0.000999 \quad \text{or} \quad \frac{1}{1001}$$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

A fair die is rolled 7 times.
Find the probability of getting

a no fives,

b only 3 fives,

c 4 fives and 3 sixes.

Solution:

a $\left(\frac{5}{6}\right)^7 = 0.279$

b

$$\left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^4 \times \frac{7!}{4!3!}$$
$$= 0.0781$$

c

$$\left(\frac{1}{6}\right)^4 \times \left(\frac{1}{6}\right)^3 \times \frac{7!}{4!3!}$$
$$= 0.000125$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

The random variable $X \sim B(8, \frac{1}{3})$. Find

a $P(X = 2)$,

b $P(X = 5)$,

c $P(X \leq 1)$.

Solution:

a

$$P(X=2) = \binom{8}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6 = 0.273$$

b

$$P(X=5) = \binom{8}{5} \times \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^3 = 0.0683$$

c

$$\begin{aligned} P(X \leq 1) &= P(X=1) + P(X=0) \\ &= 8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^7 + \left(\frac{2}{3}\right)^8 \\ &= \left(\frac{2}{3}\right)^7 \left[\frac{8}{3} + \frac{2}{3}\right] \\ &= \left(\frac{2}{3}\right)^7 \times \frac{10}{3} \\ &= 0.195 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

The random variable $Y \sim B(6, \frac{1}{4})$. Find

a $P(Y = 3)$,

b $P(Y = 1)$,

c $P(Y \geq 5)$.

Solution:

$Y \sim B\left(6, \frac{1}{4}\right)$

a

$$P(Y = 3) = \binom{6}{3} \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^3$$
$$= 0.132$$

b

$$P(Y = 1) = \binom{6}{1} \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^5$$
$$= 0.356$$

c

$$P(Y \geq 5) = P(Y = 5) + P(Y = 6)$$
$$= 6 \times \left(\frac{1}{4}\right)^5 \times \left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)^6$$
$$= \left(\frac{1}{4}\right)^5 \left[\frac{18}{4} + \frac{1}{4}\right]$$
$$= \left(\frac{1}{4}\right)^5 \times \frac{19}{4}$$
$$= 0.00464$$

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Exercise B, Question 4

Question:

A balloon manufacturer claims that 95% of his balloons will not burst when blown up. If you have 20 of these balloons to blow up for a birthday party

a what is the probability that none of them burst when blown up?

b Find the probability that exactly 2 balloons burst.

Solution:

a

$$\begin{aligned} \mathrm{P}(X=0) &= (0.95)^{20} \\ &= 0.358 \end{aligned}$$

b Let $X =$ number of balloon that *do* burst

$X \sim \mathrm{B}(20, 0.05)$

$$\begin{aligned} \mathrm{P}(X=2) &= \binom{20}{2}(0.95)^{18}(0.05)^2 \\ &= 0.189 \end{aligned}$$

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Exercise B, Question 5

Question:

A student suggests using a binomial distribution to model the following situations. Give a description of the random variable, state any assumptions that must be made and give possible values for n and p.

a A sample of 20 bolts is checked for defects from a large batch. The production process should produce 1% of defective bolts.

b Some traffic lights have three phases: stop 48% of the time, wait or get ready 4% of the time and go 48% of the time. Assuming that you only cross a traffic light when it is in the go position, model the number of times that you have to wait or stop on a journey passing through 6 sets of traffic lights.

c When Stephanie plays tennis with Timothy on average one in eight of her serves is an 'ace'. How many 'aces' does Stephanie serve in the next 30 serves against Timothy?

Solution:

a $X =$ number of defective bolts in a sample of 20

$X \sim \text{B}(20, 0.01)$

$n = 20$

$p = 0.01$

Assume bolts being defective are independent of each other

b $X =$ number of times wait or stop in 6 lights

$X \sim \text{B}(6, 0.52)$

$n = 6$

$p = 0.52.$

Assume the lights operate independently and the time lights are on/off is constant.

c $X =$ no. of aces Stephanie serves in the next 30

$X \sim \text{B}(30, \frac{1}{8})$

$n = 30$

$p = \frac{1}{8}$

Assume serves being an ace are independent and probability of an ace is constant.

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Exercise B, Question 6

Question:

State which of the following can be modelled with a binomial distribution and which can not. Give reasons for your answers.

a Given that 15% of people have blood that is Rhesus negative (Rh^-), model the number of pupils in a statistics class of 14 who are Rh^-.

b You are given a fair coin and told to keep tossing it until you obtain 4 heads in succession. Model the number of tosses you need.

c A certain car manufacturer produces 12% of new cars in the colour red, 8% in blue, 15% in white and the rest in other colours. You make a note of the colour of the first 15 new cars of this make. Model the number of red cars you observe.

Solution:

a $X =$ number of people in class of 14 who are Rh^-
$X \sim \text{B}(14, 0.15)$ is OK if we assume the children in the class being Rh^- is independent from child to child (so no siblings/twins)

b This is not binomial since the number of tosses is not fixed.
The probability of a head at each toss is constant $(p = 0.5)$ but there is no value of n.

c Assuming the colours of the cars are independent (which should be reasonable)
$X =$ number of red cars out of 15
$X \sim \text{B}(15, 0.12)$

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Exercise B, Question 7

Question:

A fair die is rolled repeatedly. Find the probability that

a the first 6 occurs on the fourth roll,

b there are 3 sixes in the first 10 rolls.

Solution:

a Sequence must be $\bar{6}\,\bar{6}\,\bar{6}\,6$

$$\text{Probability} = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$$
$$= \frac{125}{1296} \quad \text{or} \quad 0.0965$$

b $X = \text{number of sixes in 10 rolls}$

$X \sim \mathrm{B}(10, \frac{1}{6})$

$$\mathrm{P}(X = 3) = \binom{10}{3}\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^7$$
$$= 0.155$$

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Exercise B, Question 8

Question:

A coin is biased so that the probability of it landing on heads is $\frac{2}{3}$. The coin is tossed repeatedly. Find the probability that

a the first tail will occur on the fifth toss,

b in the first 7 tosses there will be exactly 2 tails.

Solution:

a Sequence must be: H H H H T
So not Binomial.

Probability $= \frac{1}{3} \times \left(\frac{2}{3}\right)^4 = \frac{16}{243}$ or 0.0658

b $X =$ number of tails in 7 tosses

$X \sim B(7, \frac{1}{3})$

$$P(X = 2) = \binom{7}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^5$$

$$= 0.307 \text{ or } \frac{224}{729}$$

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Exercise C, Question 1

Question:

The random variable $X \sim \mathrm{B}(9, 0.2)$. Find

a $\mathrm{P}(X \leq 4)$,

b $\mathrm{P}(X < 3)$,

c $\mathrm{P}(X \geq 2)$,

d $\mathrm{P}(X = 1)$.

Solution:

$X \sim \mathrm{B}(9, 0.2)$

a $\mathrm{P}(X \leq 4) = 0.9804$ (tables)

b
$$\begin{aligned} \mathrm{P}(X < 3) &= \mathrm{P}(X \leq 2) \\ &= 0.7382 \end{aligned}$$ (tables)

c
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - 0.4362 \quad \text{(tables)} \\ &= 0.5638 \end{aligned}$$

d
$$\begin{aligned} \mathrm{P}(X = 1) &= \mathrm{P}(X \leq 1) - \mathrm{P}(X = 0) \\ &= 0.4362 - 0.1342 \quad \text{(tables)} \\ &= 0.3020 \end{aligned}$$

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Exercise C, Question 2

Question:

The random variable $X \sim B(20, 0.35)$. Find

a $P(X \leq 10)$,

b $P(X > 6)$,

c $P(X = 5)$,

d $P(2 \leq X \leq 7)$.

Solution:

$X \sim B(20, 0.35)$

a $P(X \leq 10) = 0.9468$ (tables)

b
$$\begin{aligned} P(X > 6) &= 1 - P(X \leq 6) \\ &= 1 - 0.4166 \quad \text{(tables)} \\ &= 0.5834 \end{aligned}$$

c
$$\begin{aligned} P(X = 5) &= P(X \leq 5) - P(X \leq 4) \\ &= 0.2454 - 0.1182 \\ &= 0.1272 \end{aligned}$$

d
$$\begin{aligned} P(2 \leq X \leq 7) &= P(X \leq 7) - P(X \leq 1) \\ &= 0.6010 - 0.0021 \\ &= 0.5989 \end{aligned}$$

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Exercise C, Question 3

Question:

The random variable $X \sim \mathrm{B}(40, 0.45)$. Find

a $\mathrm{P}(X < 20)$,

b $\mathrm{P}(X > 16)$,

c $\mathrm{P}(11 \leq X \leq 15)$,

d $\mathrm{P}(10 < X < 17)$.

Solution:

$X \sim \mathrm{B}(40, 0.45)$

a $\mathrm{P}(X < 20) = \mathrm{P}(X \leq 19)$
$= 0.6844$ (tables)

b $\mathrm{P}(X > 16) = 1 - \mathrm{P}(X \leq 16)$
$= 1 - 0.3185$ (tables)
$= 0.6815$

c $\mathrm{P}(11 \leq X \leq 15) = \mathrm{P}(X \leq 15) - \mathrm{P}(X \leq 10)$
$= 0.2142 - 0.0074$
$= 0.2068$

d $\mathrm{P}(10 < X < 17) = \mathrm{P}(X \leq 16) - \mathrm{P}(X \leq 10)$
$= 0.3185 - 0.0074$
$= 0.3111$

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Exercise C, Question 4

Question:

The random variable $X \sim B(30, 0.15)$. Find

a $P(X > 8)$,

b $P(X \leq 4)$,

c $P(2 \leq X < 10)$,

d $P(X = 4)$.

Solution:

$X \sim B(30, 0.15)$

a $P(X > 8) = 1 - P(X \leq 8)$
$= 1 - 0.9722$ (tables)
$= 0.0278$

b $P(X \leq 4) = 0.5245$ (tables)

c $P(2 \leq X < 10) = P(X \leq 9) - P(X \leq 1)$
$= 0.9903 - 0.0480$ (tables)
$= 0.9423$

d $P(X = 4) = P(X \leq 4) - P(X \leq 3)$
$= 0.5245 - 0.3217$ (tables)
$= 0.2028$

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Exercise C, Question 5

Question:

Eight fair coins are tossed and the total number of heads showing is recorded.
Find the probability of

a no heads,

b at least 2 heads,

c more heads than tails.

Solution:

X = number of heads (coins are fair so $p = 0.5$)

$X \sim \mathrm{B}(8, 0.5)$

a $\mathrm{P}(X = 0) = (0.5)^8 = 0.0039$ (tables)

b
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - 0.0352 \quad \text{(tables)} \\ &= 0.9648 \end{aligned}$$

c
$$\begin{aligned} \mathrm{P}(X \geq 5) &= 1 - \mathrm{P}(X \leq 4) \\ &= 1 - 0.6367 \quad \text{(tables)} \\ &= 0.3633 \end{aligned}$$

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Exercise C, Question 6

Question:

For a particular type of plant 25% have blue flowers. A garden centre sells these plants in trays of 15 plants of mixed colours. A tray is selected at random.
Find the probability that the number of blue flowers this tray contains is

a exactly 4,

b at most 3,

c between 3 and 6 (inclusive).

Solution:

$X =$ number of plants with blue flowers on tray of 15
$X \sim \text{B}(15, 0.25)$

a $\text{P}(X = 4) = \text{P}(X \le 4) - \text{P}(X \le 3)$
$= 0.6865 - 0.4613$ (tables)
$= 0.2252$

b $\text{P}(X \le 3) = 0.4613$ (tables)

c $\text{P}(3 \le X \le 6) = \text{P}(X \le 6) - \text{P}(X \le 2)$
$= 0.9434 - 0.2361$ (tables)
$= 0.7073$

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Exercise C, Question 7

Question:

The random variable $X \sim B(50, 0.40)$. Find

a the largest value of k such that $P(X \le k) < 0.05$,

b the smallest number r such that $P(X > r) < 0.01$.

Solution:

$X \sim B(50, 0.40)$

a $P(X \le 13) = 0.0280$ (tables)

$P(X \le 14) = 0.0540$

$\therefore k = 13$

b $P(X \le 27) = 0.9840 \Rightarrow P(X > 27) = 0.0160 > 0.01$

$P(X \le 28) = 0.9924 \Rightarrow P(X > 28) = 0.0076 < 0.01$

$\therefore r = 28$

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Exercise C, Question 8

Question:

The random variable $X \sim \mathrm{B}(40, 0.10)$. Find

a the largest value of k such that $\mathrm{P}(X < k) < 0.02$,

b the smallest number r such that $\mathrm{P}(X > r) < 0.01$,

c $\mathrm{P}(k \leq X \leq r)$.

Solution:

$X \sim \mathrm{B}(40, 0.10)$

a
$$\begin{aligned} \mathrm{P}(X=0) &= 0.0148 < 0.02 \\ \mathrm{P}(X \leq 1) &= 0.0805 > 0.02 \qquad \text{(tables)} \\ \therefore \mathrm{P}(X < 1) &= 0.0148 < 0.02 \text{ and so } k = 1 \end{aligned}$$

b
$$\begin{aligned} \mathrm{P}(X \leq 8) &= 0.9845 \Rightarrow \mathrm{P}(X > 8) = 0.0155 > 0.01 \qquad \text{(tables)} \\ \mathrm{P}(X \leq 9) &= 0.9949 \Rightarrow \mathrm{P}(X > 9) = 0.0051 < 0.01 \\ r &= 9 \end{aligned}$$

c
$$\begin{aligned} \mathrm{P}(k \leq X \leq r) &= \mathrm{P}(X \leq r) - \mathrm{P}(X \leq k-1) \\ &= \mathrm{P}(X \leq 9) - \mathrm{P}(X = 0) \\ &= 0.9949 - 0.0148 \\ &= 0.9801 \end{aligned}$$

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Exercise C, Question 9

Question:

In a town, 30% of residents listen to the local radio.
Ten residents are chosen at random.

a State the distribution of the random variable
$X =$ the number of these 10 residents that listen to the local radio.

b Find the probability that at least half of these 10 residents listen to local radio.

c Find the smallest value of s so that $P(X \geq s) < 0.01$.

Solution:

$X =$ number of residents out of 10 who listen to local radio

a $X \sim B(10, 0.30)$

b
$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.8497 \quad \text{(tables)} \\ &= 0.1503 \end{aligned}$$

c
$$\begin{aligned} P(X \leq 6) &= 0.9894 \text{ so } P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01 \\ P(X \leq 7) &= 0.9984 \\ P(X \geq 8) &= 1 - 0.9984 \quad \text{(tables)} \\ &= 0.0016 \quad < 0.01 \\ s &= 8 \end{aligned}$$

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Exercise C, Question 10

Question:

A factory produces a component for the motor trade and 5% of the components are defective. A quality control officer regularly inspects a random sample of 50 components. Find the probability that the next sample contains

a fewer than 2 defectives,

b more than 5 defectives.

The officer will stop production if the number of defectives in the sample is greater than a certain value d. Given that the officer stops production less than 5% of the time,

c find the smallest value of d.

Solution:

X = number of defects in 50 components
$X \sim \text{B}(50, 0.05)$

a $\text{P}(X < 2) = \text{P}(X \le 1)$ (tables)
$= 0.2794$

b $\text{P}(X > 5) = 1 - \text{P}(X \le 5)$
$= 1 - 0.9622$
$= 0.0378$

c Seek d such that

$\text{P}(X > d) < 0.05$
$\text{P}(X \le 4) = 0.8964 \Rightarrow \text{P}(X > 4) = 0.1036 > 0.05$
$\text{P}(X \le 5) = 0.9622 \Rightarrow \text{P}(X > 5) = 0.0378 < 0.05$
$\therefore d = 5$

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Exercise D, Question 1

Question:

A fair cubical die is rolled 36 times and the random variable X represents the number of sixes obtained. Find the mean and variance of X.

Solution:

$X \sim \mathrm{B}(36, \frac{1}{6})$ fair die $\Rightarrow p = \frac{1}{6}$

$$\mathrm{E}(X) = 36 \times \frac{1}{6} = 6$$

$$\mathrm{Var}(X) = 36 \times \frac{1}{6} \times \frac{5}{6} = \frac{30}{6} = 5$$

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Exercise D, Question 2

Question:

a Find the mean and variance of the random variable $X \sim B(12, 0.25)$.

b Find $P(\mu - \sigma < X < \mu + \sigma)$.

Solution:

a $X \sim B(12, 0.25)$

$$\begin{aligned} E(X) &= 12 \times 0.25 = 3 \\ Var(X) &= 12 \times 0.25 \times 0.75 = 3 \times \frac{3}{4} = \frac{9}{4} \text{ or } 2.25 \end{aligned}$$

b $\sigma^2 = \frac{9}{4} \Rightarrow \sigma = \frac{3}{2}$ or 1.5

$$\begin{aligned} & P(\mu - \sigma < X < \mu + \sigma) \\ = \; & P\left(3 - \frac{3}{2} < X < 3 + \frac{3}{2}\right) \\ = \; & P\left(\frac{3}{2} < X < 4\frac{1}{2}\right) \\ = \; & P(X \le 4) - P(X \le 1) \\ = \; & 0.8424 - 0.1584 \qquad \text{(tables)} \\ = \; & 0.6840 \end{aligned}$$

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Exercise D, Question 3

Question:

a Find the mean and variance of the random variable $X \sim B(30, 0.40)$.

b Find $P(\mu - \sigma < X \leq \mu)$.

Solution:

$X \sim B(30, 0.40)$

a

$$E(X) = 30 \times 0.40 = 12$$
$$Var(X) = 30 \times 0.40 \times 0.6 = 12 \times 0.6 = 7.2$$

b $\sigma^2 = 7.2 \Rightarrow \sigma = 2.68\ldots$

$P(12 - 2.68\ldots < X \leq 12)$

$= P(9.32\ldots < X \leq 12)$

$= P(X \leq 12) - P(X \leq 9)$

$= 0.5785 - 0.1763$ (tables)

$= 0.4022$

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Exercise D, Question 4

Question:

It is estimated that 1 in 20 people are left-handed.

a What size sample should be taken to ensure that the expected number of left-handed people in the sample is 3?

b What is the standard deviation of the number of left-handed people in this case?

Solution:

X = number of left-handed people in a sample of size n

$X \sim \mathrm{B}(n, \frac{1}{20})$

a

$$\mathrm{E}(X) = 3 \Rightarrow \frac{n}{20} = 3$$
$$\therefore n = 60$$

b

$$\mathrm{Var}(X) = 60 \times \frac{1}{20} \times \frac{19}{20} = 2.85$$
$$\therefore \sigma^2 = 2.85$$
$$\sigma = \sqrt{2.85} = 1.69$$

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Exercise D, Question 5

Question:

An experiment is conducted with a fair die to examine the number of sixes that occur. It is required to have the standard deviation smaller than 1. What is the largest number of throws that can be made?

Solution:

$X =$ number of sixes in n throws

$X \sim \mathrm{B}(n, \frac{1}{6})$

$\mathrm{Var}(X) = n \times \frac{1}{6} \times \frac{5}{6} = \frac{5n}{36}$

standard deviation < 1

$$\Rightarrow \frac{\sqrt{5n}}{6} < 1$$

$$\sqrt{5n} < 6$$

$$5n < 36$$

$$n < \frac{36}{5} = 7.2$$

$\therefore$ need $n = 7$

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Exercise D, Question 6

Question:

The random variable $X \sim \mathrm{B}(n, p)$ has a mean of 45 and standard deviation of 6. Find the value of n and the value of p.

Solution:

$X \sim \mathrm{B}(n, p)$

$$\begin{aligned}
\mathrm{E}(X) = 45 \Rightarrow np &= 45 \\
\sigma = 6 \Rightarrow \mathrm{Var}(X) = 36 \Rightarrow np(1-p) &= 36 \\
\therefore 45(1-p) &= 36 \\
\therefore 1-p &= \frac{36}{45} \\
p &= 1-\frac{36}{45} \\
p &= \frac{9}{45} = \frac{1}{5} \\
\text{and } n &= 225
\end{aligned}$$

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Exercise E, Question 1

Question:

A coin is biased so that the probability of a head is $\frac{2}{3}$. The coin is tossed repeatedly.

Find the probability that

a the first tail will occur on the sixth toss,

b in the first 8 tosses there will be exactly 2 tails.

Solution:

a Sequence is: H H H H H T

probability : $\left(\frac{2}{3}\right)^5 \times \frac{1}{3} = 0.0439$ or $\frac{32}{729}$

b $X =$ number of tails in 8 tosses

$X \sim \mathrm{B}(8, \frac{1}{3})$

$$\mathrm{P}(X = 2) = \binom{8}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6$$

$$= 0.273$$

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Exercise E, Question 2

Question:

Records kept in a hospital show that 3 out of every 10 patients who visit the accident and emergency department have to wait more than half an hour. Find, to 3 decimal places, the probability that of the first 12 patients who come to the accident and emergency department

a none,

b more than 2,

will have to wait more than half an hour.

Solution:

$X =$ number of patients waiting more than $\frac{1}{2}$ hour.

$X \sim \mathrm{B}(12, 0.3)$

a $\mathrm{P}(X = 0) = (0.7)^{12} = 0.01384\ldots$

$= 0.0138 \quad (3\text{ s.f.})$

b $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$

$= 1 - 0.2528 \quad \text{(tables)}$

$= 0.7472$

$= 0.747 \quad (3\text{ d.p.})$

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Exercise E, Question 3

Question:

A factory is considering two methods of checking the quality of production of the batches of items it produces.

Method I A random sample of 10 items is taken from a large batch and the batch is accepted if there are no defectives in this sample. If there are 2 or more defectives the batch is rejected. If there is only 1 defective then another sample of 10 is taken and the batch is accepted if there are no defectives in this second sample, otherwise the whole batch is rejected.

Method II A random sample of 20 items is taken from a large batch and the batch is accepted if there is at most 1 defective in this sample, otherwise the whole batch is rejected.

The factory knows that 1% of items produced are defective and wishes to use the method of checking the quality of production for which the probability of accepting the whole batch is largest.

a Decide which method the factory should use.

b Determine the expected number of items sampled using Method I.

Solution:

a X = number of defective items in sample of 10

Method I
$X \sim B(10, 0.01)$

$$\begin{aligned} P(\text{Accept}) &= P(X=0)+P(X=1)\times P(X=0) \\ &= P(X=0)[1+P(X=1)] \\ &= (0.99)^{10}[1+10\times 0.01\times (0.99)^9] \\ &= 0.98699\ldots \quad = 0.987 \end{aligned}$$

or from tables

$$\begin{aligned} P(\text{Accept}) &= P(X=0)+P(X=1)\times P(X=0) \\ &= 0.9044+[0.9957-0.9044]\times 0.9044 \\ &= 0.98697 \\ &= 0.987 \end{aligned}$$

Method II Y = number of defective items in sample of 20
$Y \sim B(20, 0.01)$

$$\begin{aligned} P(\text{Accept}) &= P(Y \le 1) \\ &= (0.99)^{20}+20\times (0.01)\times (0.99)^{19} \text{ or find direct from tables} \\ &= 0.98314\ldots \quad = 0.983 \end{aligned}$$

$\therefore$ use Method I

b *Method I*

Number sampled	10	20
probability	$1-P(X=1)$	$P(X=1)$

$$\begin{aligned} \therefore \text{expected number} &= 10(1-P(X=1))+20P(X=1) \\ &= 10+10\times P(X=1) \\ &= 10+0.9135\ldots \\ &= 10.9 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

Question:

a State clearly the conditions under which it is appropriate to assume that a random variable has a binomial distribution.

A door-to-door canvasser tries to persuade people to have a certain type of double glazing installed. The probability that his canvassing at a house is successful is 0.05.

b Find the probability that he will have at least 2 successes out of the first 10 houses he canvasses.

c Find the number of houses he should canvass per day in order to average 3 successes per day.

d Calculate the least number of houses that he must canvass in order that the probability of his getting at least one success exceeds 0.99. ***E***

Solution:

a
1. There are n independent trials.
2. n is a fixed number.
3. The outcome of each trial is success or failure.
4. The probability of success at each trial is constant.
5. The outcome of any trial is independent of any other trial.

b $X =$ number of successes

$X \sim \text{B}(10, 0.05)$

$$\begin{aligned} \text{P}(X \geq 2) &= 1 - \text{P}(X \leq 1) \\ &= 1 - 0.9139 \qquad \text{(tables)} \\ &= 0.0861 \end{aligned}$$

c $Y =$ number of successes in n houses

$Y \sim \text{B}(n, 0.05)$

$n \times 0.05 = 3$

$\text{E}(Y) = 3 \Rightarrow \text{or } \frac{n}{20} = 3 \quad \therefore n = 60$

d

$$\begin{aligned} \text{P}(Y \geq 1) &> 0.99 \\ \Rightarrow 1 - \text{P}(Y = 0) &> 0.99 \\ 0.01 &> \text{P}(Y = 0) \\ 0.01 &> (0.95)^n \\ \log(0.01) &> n\log(0.95) \\ \frac{\log 0.01}{\log 0.95} &< n \quad \leftarrow [\log 0.95 < 0 \text{ so change } > \text{ when dividing}] \\ n &> 89.78\ldots \text{ so } n = 90 \end{aligned}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 5

Question:

An archer fires arrows at a target and for each arrow, independently of all the others, the probability that it hits the bull's eye is $\frac{1}{8}$.

a Given that the archer fires 5 arrows, find the probability that fewer than 2 arrows hit the bull's-eye.

The archer fires 5 arrows, collects them from a target and fires all 5 again.

b Find the probability that on both occasions fewer than 2 hit the bull's eye.

Solution:

$X =$ number of bull's eyes in 5 arrows

$X \sim \mathrm{B}(5, \frac{1}{8})$

a
$$\begin{aligned} \mathrm{P}(X < 2) &= \mathrm{P}(X \le 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) \\ &= \left(\frac{7}{8}\right)^5 + 5 \times \frac{1}{8} \times \left(\frac{7}{8}\right)^4 \\ &= 0.87927 \\ &= 0.879 \end{aligned}$$

b
$$\begin{aligned} [\mathrm{P}(X < 2)]^2 &= 0.87927^2 \\ &= 0.773 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise E, Question 6

Question:

A completely unprepared student is given a true/false type test with 10 questions. Assuming that the student answers all the questions at random

a find the probability that the student gets all the answers correct.

It is decided that a pass will be awarded for 8 or more correct answers.

b Find the probability that the student passes the test.

Solution:

X = number of correctly answered questions

$X \sim B(10, 0.5)$

a

$$P(X=10)=(0.5)^{10} = 0.00097656\ldots$$
$$= 0.000977 \quad (3\text{ s.f.})$$

b

$$P(X \geq 8) = 1 - P(X \leq 7)$$
$$= 1 - 0.9453 \quad \text{(tables)}$$
$$= 0.0547$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 7

Question:

A six-sided die is biased. When the die is thrown the number 5 is twice as likely to appear as any other number. All the other faces are equally likely to appear. The die is thrown repeatedly. Find the probability that

a the first 5 will occur on the sixth throw,

b in the first eight throws there will be exactly three 5s. ***E***

Solution:

x	1	2	3	4	5	6
$\mathrm{P}(X=x)$	p	P	P	P	$2p$	p

$7p = 1 \Rightarrow p = \frac{1}{7}$

a Not binomial

Sequence: $\bar{5}\,\bar{5}\,\bar{5}\,\bar{5}\,\bar{5}\,5$

Probability: $\left(\frac{5}{7}\right)^5 \left(\frac{2}{7}\right)$

$= 0.0531$

b $Y =$ number of 5s in 8 throws

$Y \sim \mathrm{B}(8, \frac{2}{7})$

$$\mathrm{P}(Y=3) = \binom{8}{3} \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^5$$

$= 0.24285\ldots$

$= 0.243 \quad (3\text{ s.f.})$

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Exercise E, Question 8

Question:

A manufacturer produces large quantities of plastic chairs. It is known from previous records that 15% of these chairs are green. A random sample of 10 chairs is taken.

a Define a suitable distribution to model the number of green chairs in this sample.

b Find the probability of at least 5 green chairs in this sample.

c Find the probability of exactly 2 green chairs in this sample.

Solution:

X = number of green chairs in sample of 10

a $X \sim \text{B}(10, 0.15)$

b

$$\begin{aligned} \text{P}(X \geq 5) &= 1 - \text{P}(X \leq 4) \\ &= 1 - 0.9901 \qquad \text{(tables)} \\ &= 0.0099 \end{aligned}$$

c

$$\begin{aligned} \text{P}(X = 2) &= \text{P}(X \leq 2) - \text{P}(X \leq 1) \\ &= 0.8202 - 0.5443 \qquad \text{(tables)} \\ &= 0.2759 \end{aligned}$$

Solutionbank S2

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Exercise E, Question 9

Question:

A bag contains a large number of beads of which 45% are yellow. A random sample of 20 beads is taken from the bag. Use the binomial distribution to find the probability that the sample contains

a fewer than 12 yellow beads,

b exactly 12 yellow beads. ***E***

Solution:

$X =$ number of yellow beads in sample of 20

$X \sim \text{B}(20, 0.45)$

a

$$\begin{aligned} \text{P}(X < 12) &= \text{P}(X \leq 11) \\ &= 0.8692 \end{aligned}$$

b

$$\begin{aligned} \text{P}(X = 12) &= \text{P}(X \leq 12) - \text{P}(X \leq 11) \\ &= 0.9420 - 0.8692 \\ &= 0.0728 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The discrete random variable $X \sim \text{Po}(2.3)$. Find

a $\text{P}(X = 4)$,

b $\text{P}(X \geq 1)$,

c $\text{P}(4 < X < 6)$.

Solution:

$X \sim \text{P}_0(2.3)$

a $\text{P}(X = 4) = \dfrac{e^{-2.3}(2.3)^4}{4!} = 0.11690\ldots = 0.117 \text{ (3 s.f.)}$

b
$$\begin{aligned}\text{P}(X \geq 1) &= 1 - \text{P}(X = 0) \\ &= 1 - e^{-2.3} \\ &= 0.89974\ldots = 0.900 \text{ (3 s.f.)}\end{aligned}$$

c $\text{P}(4 < X < 6)$

$= \text{P}(X = 5)$

$= 0.05377 \quad = 0.0538 \text{ (3 s.f.)}$

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Exercise A, Question 2

Question:

The discrete random variable $X \sim \text{Po}(5.7)$. Find

a $\text{P}(X=7)$,

b $\text{P}(X \le 1)$,

c $\text{P}(X > 2)$.

Solution:

$X \sim \text{P}_0(5.7)$

a
$$\text{P}(X=7) = \frac{e^{-5.7}(5.7)^7}{7!} = 0.12978\ldots = 0.130 \text{ (3 s.f.)}$$

b
$$\begin{aligned}\text{P}(X \le 1) &= \text{P}(X=0)+\text{P}(X=1)\\ &= e^{-5.7} + e^{-5.7} \times \frac{5.7}{1!}\\ &= 6.7 \times e^{-5.7}\\ &= 0.022417\ldots = 0.0224 \text{ (3 s.f.)}\end{aligned}$$

c
$$\begin{aligned}\text{P}(X > 2) &= 1-\text{P}(X \le 2)\\ &= 1-[\text{P}(X=0)+\text{P}(X=1)+\text{P}(X=2)]\\ &= 1-0.022417-\frac{e^{-5.7} \times 5.7^2}{2!}\\ &= 0.923226\ldots = 0.923 \text{ (3 s.f.)}\end{aligned}$$

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Exercise A, Question 3

Question:

The random variable $Y \sim \text{Po}(0.35)$. Find

a $\text{P}(Y=1)$,

b $\text{P}(Y \geq 1)$,

c $\text{P}(1 \leq Y < 3)$.

Solution:

$Y \sim \text{P}_0(0.35)$

a
$$\begin{aligned} \text{P}(Y=1) &= \frac{e^{-0.35} \times 0.35}{1!} \\ &= 0.24664\ldots \quad = 0.247 \end{aligned}$$

b
$$\begin{aligned} \text{P}(Y \geq 1) &= 1 - \text{P}(Y=0) \\ &= 0.29531\ldots \quad = 0.295 \end{aligned}$$

c
$$\begin{aligned} \text{P}(1 \leq Y < 3) &= \text{P}(Y=1) + \text{P}(Y=2) \\ &= 0.24664\ldots + \frac{e^{-0.35} \times (0.35)^2}{2!} \\ &= 0.289802\ldots \quad = 0.290 \end{aligned}$$

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Exercise A, Question 4

Question:

The random variable $X \sim \text{Po}(3.6)$. Find

a $\text{P}(X = 5)$,

b $\text{P}(3 < X \leq 6)$,

c $\text{P}(X < 2)$.

Solution:

$X \sim \text{P}_0(3.6)$

a

$$\text{P}(X = 5) = \frac{e^{-3.6}(3.6)^5}{5!} = 0.13768\ldots = 0.138$$

b

$$\begin{aligned}\text{P}(3 < X \leq 6) &= \text{P}(X = 4) + \text{P}(X = 5) + \text{P}(X = 6) \\ &= \frac{e^{-3.6}(3.6)^4}{4!} + \frac{e^{-3.6}(3.6)^5}{5!} + \frac{e^{-3.6}(3.6)^6}{6!} \\ &= 0.41151\ldots = 0.412\end{aligned}$$

c

$$\begin{aligned}\text{P}(X < 2) &= \text{P}(X = 0) + \text{P}(X = 1) \\ &= e^{-3.6} + e^{-3.6} \times 3.6 \\ &= 4.6 \times e^{-3.6} \\ &= 0.12568\ldots = 0.126\end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

The random variable $X \sim \text{Po}(2.5)$. Find

a $\text{P}(X=1)$,

b $\text{P}(X>2)$,

c $\text{P}(X \le 5)$,

d $\text{P}(3 \le X \le 5)$.

Solution:

$X \sim \text{P}_0(2.5)$

a $\text{P}(X=1) = e^{-2.5} \times 2.5 \quad = 0.2052$ (4 d.p.)

b
$$\begin{aligned} \text{P}(X>2) &= 1-\text{P}(X \le 2) \\ &= 1-0.5438 \quad \text{(tables)} \\ &= 0.4562 \end{aligned}$$

c $\text{P}(X \le 5) = 0.9580$ (tables)

d
$$\begin{aligned} \text{P}(3 \le X \le 5) &= \text{P}(X \le 5) - \text{P}(X \le 2) \\ &= 0.9580 - 0.5438 \quad \text{(tables)} \\ &= 0.4142 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

The random variable $X \sim \text{Po}(6)$. Find

a $P(X \leq 3)$,

b $P(X > 4)$,

c $P(X = 5)$,

d $P(2 < X \leq 7)$.

Solution:

$X \sim P_0(6)$

a $P(X \leq 3) = 0.1512$ (tables)

b
$$\begin{aligned} P(X > 4) &= 1 - P(X \leq 4) \\ &= 1 - 0.2851 \quad \text{(tables)} \\ &= 0.7149 \end{aligned}$$

c
$$\begin{aligned} P(X = 5) &= P(X \leq 5) - P(X \leq 4) \\ &= 0.4457 - 0.2851 \quad \text{(tables)} \\ &= 0.1606 \end{aligned}$$

d
$$\begin{aligned} P(2 < X \leq 7) &= P(X \leq 7) - P(X \leq 2) \\ &= 0.7440 - 0.0620 \quad \text{(tables)} \\ &= 0.6820 \end{aligned}$$

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Exercise B, Question 3

Question:

The random variable Y has a Poisson distribution with mean 4.5. Find

a $P(Y = 2)$,

b $P(Y \le 1)$,

c $P(Y > 4)$,

d $P(2 \le Y \le 6)$.

Solution:

$Y \sim P_0(4.5)$

a $P(Y = 2) = P(Y \le 2) - P(Y \le 1)$

$= 0.1736 - 0.0611$ (tables)

$= 0.1125$

b $P(Y \le 1) = 0.0611$ (tables)

c $P(Y > 4) = 1 - P(Y \le 4)$

$= 1 - 0.5321$ (tables)

$= 0.4679$

d $P(2 \le Y \le 6) = P(Y \le 6) - P(Y \le 1)$

$= 0.8311 - 0.0611$ (tables)

$= 0.7700$

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Exercise B, Question 4

Question:

The random variable $X \sim \text{Po}(8)$. Find the values of a, b, c and d such that

a $\text{P}(X \le a) = 0.3134$,

b $\text{P}(X \le b) = 0.7166$,

c $\text{P}(X < c) = 0.0996$,

d $\text{P}(X > d) = 0.8088$.

Solution:

$X \sim \text{P}_0(8)$

a
$$\begin{aligned} \text{P}(X \le 6) &= 0.3134 \\ \therefore a &= 6 \end{aligned} \quad \text{(tables)}$$

b
$$\begin{aligned} \text{P}(X \le 9) &= 0.7166 \\ \therefore b &= 9 \end{aligned} \quad \text{(tables)}$$

c
$$\begin{aligned} \text{P}(X \le 4) &= 0.0996 \\ \text{P}(X < 5) &= 0.0996 \\ \therefore c &= 5 \end{aligned} \quad \text{(tables)}$$

d
$$\begin{aligned} \text{P}(X \le 5) &= 0.1912 \\ \therefore \text{P}(X > 5) &= 1 - 0.1912 \\ &= 0.8088 \\ \therefore d &= 5 \end{aligned} \quad \text{(tables)}$$

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Exercise B, Question 5

Question:

The random variable $X \sim \text{Po}(3.5)$. Find the values of a, b, c and d such that

a $\text{P}(X \le a) = 0.8576$,

b $\text{P}(X > b) = 0.6792$,

c $\text{P}(X \le c) \ge 0.95$,

d $\text{P}(X > d) \le 0.005$.

Solution:

$X \sim \text{P}_0(3.5)$

a
$$\text{P}(X \le 5) = 0.8576 \quad \text{(tables)}$$
$$\therefore a = 5$$

b
$$1 - 0.6792 = 0.3208$$
$$\text{P}(X \le 2) = 0.3208 \quad \text{(tables)}$$
$$\text{P}(X > 2) = 0.6792$$
$$\therefore b = 2$$

c
$$\text{P}(X \le 6) = 0.9347$$
$$\text{P}(X \le 7) = 0.9733 \quad \text{(tables)}$$
$$\therefore c \ge 7$$

d
$$\text{P}(X \le 8) = 0.9901 \Rightarrow \text{P}(X > 8) = 0.0099 > 0.005$$
$$\text{P}(X \le 9) = 0.9967 \Rightarrow \text{P}(X > 9) = 0.0033 < 0.005 \quad \text{(tables)}$$
$$\therefore d \ge 9$$

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Exercise B, Question 6

Question:

The number of telephone calls received at an exchange during a weekday morning follows a Poisson distribution with a mean of 6 calls per 5-minute period. Find the probability that

a there are no calls in the next 5 minutes,

b 3 or fewer calls are received in the next 5 minutes,

c fewer than 2 calls are received between 11:00 and 11:05,

d no more than 2 calls are received between 11:30 and 11:35.

Solution:

Let X = number of telephone calls in a week day morning in 5 minutes

$X \sim \text{P}_0(6)$

a $\text{P}(X = 0) = e^{-6} = 0.002478$

$= 0.00248$ (3 s.f.)

or 0.0025 (tables)

b $\text{P}(X \leq 3) = 0.1512$ (tables)

c $\text{P}(X < 2) = \text{P}(X \leq 1)$ (tables)

$= 0.0174$

d $\text{P}(X \leq 2) = 0.0620$ (tables)

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

The random variable $X \sim \text{Po}(9)$. Find

a $\mu = \text{E}(X)$,

b σ = standard deviation of X,

c $\text{P}(\mu \le X < \mu + \sigma)$,

d $\text{P}(X \le \mu - \sigma)$.

Solution:

$X \sim \text{P}_0(9)$

a $\mu = \text{E}(X) = 9$

b $\sigma^2 = \text{Var}(X) = 9 \quad \sigma = 3$

c
$$\begin{aligned} \text{P}(9 \le X < 12) &= \text{P}(X \le 11) - \text{P}(X \le 8) \\ &= 0.8030 - 0.4557 \\ &= 0.3473 \end{aligned}$$

d
$$\begin{aligned} \text{P}(X \le 9 - 3) &= \text{P}(X \le 6) \qquad \text{(tables)} \\ &= 0.2068 \end{aligned}$$

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Exercise B, Question 8

Question:

The mean number of faults in 2 m^2 of cloth produced by a factory is 1.5.

a Find the probability of a 2 m^2 piece of cloth containing no faults.

b Find the probability that a 2 m^2 piece of cloth contains no more than 2 faults.

Solution:

Let X = number of faults in 2 m^2 of cloth

$X \sim \text{P}_0(1.5)$

a $\text{P}(X = 0) = 0.2231$ (tables)

b $\text{P}(X \leq 2) = 0.8088$ (tables)

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

A technician is responsible for a large number of machines. Minor adjustments have to be made to these machines and these occur at random and at a constant rate of 7 per hour. Find the probability that

a in a particular hour the technician makes 4 or fewer adjustments,

b during a half-hour break no adjustments will be required.

Solution:

Let X = number of adjustments in an hour

$X \sim \mathrm{P_0}(7)$

a $\mathrm{P}(X \leq 4) = 0.1730$ (tables)

b Y = number of adjustments in a half-hour

$Y \sim \mathrm{P_0}(3.5)$

$\mathrm{P}(Y = 0) = 0.0302$ (tables)

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Exercise C, Question 2

Question:

A textile firm produces rolls of cloth but slight defects sometimes occur. The average number of defects per square metre is 2.5. Use a Poisson distribution to calculate the probability that

a a $1.5\,\text{m}^2$ portion of cloth bought to make a skirt contains no defects,

b a $4\,\text{m}^2$ portion of cloth contains fewer than 5 defects.

c State briefly what assumptions have to be made before a Poisson distribution can be accepted as a suitable model in this situation.

Solution:

a Let X = number of defects in $1.5\,\text{m}^2$

$$\lambda = 1.5 \times 2.5 = 3.75$$

$$\therefore X \sim \text{P}_0(3.75)$$

$$\text{P}(X = 0) = e^{-3.75} = 0.0235177\ldots$$

$$= 0.0235\ (3\text{ s.f.})$$

b Let Y = number of defects in $4\,\text{m}^2$

$$\lambda = 4 \times 2.5 = 10$$

$$\therefore Y \sim \text{P}_0(10) \qquad \text{(tables)}$$

$$\text{P}(Y < 5) = \text{P}(Y \leq 4) = 0.0293$$

c Assume that defects occur independently and at random in the cloth and defects occur at a constant rate.

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Exercise C, Question 3

Question:

State which of the following could be modelled by a Poisson distribution and which can not. Give reasons for your answers.

a The number of misprints on this page in the first draft of this book.

b The number of pigs in a particular 5 m square of their field 1 hour after their food was placed in a central trough.

c The number of pigs in a particular 5 m square of their field 1 minute after their food was placed in a central trough.

d The amount of salt, in mg, contained in $1\,cm^3$ of water taken from a bucket immediately after a teaspoon of salt was added to the bucket.

e The number of marathon runners passing the finishing post between 20 and 21 minutes after the winner of the race.

Solution:

a If the misprints occur independently and at random and at a constant average rate then this could be Poisson.

b Yes because after 1 hour the pigs are probably dispersed fairly randomly and independently around the field.

c No because after 1 minute the pigs will probably be clustered around the feeding trough and so will not be randomly and independently scattered.

d No because the salt needs to diffuse so that it is randomly dissolved at a constant rate throughout the contents of the bucket.

e Yes, this may be Poisson provided that the runners are not in groups, since they need to pass the post independently and at random.

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Exercise C, Question 4

Question:

The number of accidents per week at a certain road intersection has a Poisson distribution with parameter 2.5. Find the probability that

a exactly 5 accidents will occur in a particular week,

b more than 14 accidents will occur in 4 weeks. ***E***

Solution:

a X = number of accidents in a week

$X \sim \text{P}_0(2.5)$

$\text{P}(X=5) = \text{P}(X \le 5) - \text{P}(X \le 4)$

$= 0.9580 - 0.8912$ (tables)

$= 0.0668$

b $Y =$ number of accidents in a 4-week period

$Y \sim \text{P}_0(4 \times 2.5 = 10)$

$\text{P}(Y > 14) = 1 - \text{P}(Y \le 14)$

$= 1 - 0.9165$ (tables)

$= 0.0835$

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Exercise C, Question 5

Question:

In a particular district it has been found, over a long period, that the number, X, of cases of measles reported per month has a Poisson distribution with parameter 1.5. Find the probability that in this district

a in any given month, exactly 2 cases of measles will be reported,

b in a period of 6 months, fewer than 10 cases of measles will be reported.

Solution:

$X \sim \mathrm{P_0}(1.5)$

a $\mathrm{P}(X = 2) = \mathrm{P}(X \le 2) - \mathrm{P}(X \le 1)$

$= 0.8088 - 0.5578$

$= 0.2510$

b $Y =$ number of reported cases of measles in 6 months

$Y \sim \mathrm{P_0}(9)$ ← $9 = 6 \times 1.5$

$\mathrm{P}(Y < 10) = \mathrm{P}(Y \le 9)$

$= 0.5874$ (tables)

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Exercise C, Question 6

Question:

A biologist is studying the behaviour of sheep in a large field. The field is divided into a number of equally sized squares and the average number of sheep per square is 2.5. The sheep are randomly scattered throughout the field.

a Suggest a suitable model for the number of sheep in a square and give a value for any parameter or parameters required.

b Calculate the probability that a randomly selected square contains more than 3 sheep.

A sheep dog has been sent into the field to round up the sheep.

c Explain why the model may no longer be applicable.

Solution:

a X = number of sheep per square

$X \sim \mathrm{P_0}(2.5)$

b

$$\begin{aligned} \mathrm{P}(X > 3) &= 1 - \mathrm{P}(X \le 3) \\ &= 1 - 0.7576 \quad \text{(tables)} \\ &= 0.2424 \end{aligned}$$

c The sheep will no longer be randomly scattered.

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Exercise C, Question 7

Question:

During office hours, telephone calls to a single telephone in an office come in at an average rate of 18 calls per hour. Assuming that a Poisson distribution can be applied, find the probability that in a 5-minute period there will be

a fewer than 2 calls,

b more than 3 calls.

c Find the probability of no calls during a 20-minute coffee break.

Solution:

Let X = number of calls in a 5-minute period

$\lambda = \frac{18}{12} = 1.5$

$\therefore X \sim P_0(1.5)$

a $P(X < 2) = P(X \leq 1)$

$= 0.5578$ (tables)

b $P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.9344$ (tables)

$= 0.0656$

c Y = number of calls in 20-minute period

$\lambda = 4 \times 1.5 = 6$

$\therefore Y \sim P_0(6)$

$P(Y = 0) = 0.0025$ (tables)

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Exercise C, Question 8

Question:

A shop sells large birthday cakes at a rate of 2 every 3 days.

a Find the probability of selling no large birthday cakes on a randomly selected day.

Fresh cakes are baked every 3 days and any cakes older than 3 days can not be sold.

b Find how many large birthday cakes should be baked so that the probability of running out of large birthday cakes to sell is less than 1%.

Solution:

$X =$ number of large cakes sold in a day

$X \sim \text{P}_0\left(\frac{2}{3}\right)$

a

$$\text{P}(X = 0) = e^{-\frac{2}{3}} = 0.51341$$
$$= 0.513 \,(3\text{ s.f.})$$

b $Y =$ number of large cakes sold in 3 days

$Y \sim \text{P}_0(2)$

Let $n =$ number of cakes baked

To run out of cakes you require $Y > n$

Require $\text{P}(Y > n) < 0.01$

i.e. $\text{P}(Y \le n) > 0.99$

$\text{P}(Y \le 5) = 0.9834 < 0.99$ (tables)

$\text{P}(Y \le 6) = 0.9955 > 0.99$

$\therefore$ need $n = 6$

($n \ge 6$ but baking more is likely to create more waste)

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

On a typical summer's day a boat company hires rowing boats at a rate of 9 per hour.

a Find the probability of hiring out at least 6 boats in a randomly selected 30-minute period.

The company has 8 boats to hire and decides to hire them out for 20-minute periods.

b Show that the probability of running out of boats is less than 1%.

c Find how many boats the company should have to be 99% sure of meeting all demands if the hire period is extended to 30 minutes.

Solution:

a Let X = number of boats hired in a 30-minute period

$\lambda = \frac{1}{2} \times 9 = 4.5$

$\therefore X \sim P_0(4.5)$ (tables)

$P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - 0.7029$

$= 0.2971$

b Let Y = number of requests for hire in 20-minute period

$\lambda = \frac{1}{3} \times 9 = 3$

$Y \sim P_0(3)$

$P(Y > 8) = 1 - P(Y \leq 8)$

$= 1 - 0.9962$ (tables)

$= 0.0038 < 0.01$

c Let n = number of boats

Require $P(X > n) < 0.01$

or $P(X \leq n) > 0.99$

Use $P_0(4.5)$.

$P(X \leq 9) = 0.9829 < 0.99$ (tables)

$P(X \leq 10) = 0.9933 > 0.99$

$\therefore$ they would need 10 boats

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Exercise C, Question 10

Question:

Breakdowns on a particular machine occur at random at a rate of 1.5 per week.

a Find the probability that no more than 2 breakdowns occur in a randomly chosen week.

b Find the probability of at least 5 breakdowns in a randomly chosen two-week period.

A maintenance firm offers a contract for repairing breakdowns over a six-week period. The firm will give a full refund if there are more than n breakdowns in a six-week period. The firm want the probability of having to pay a refund to be 5% or less.

c find the smallest value of n.

Solution:

Let X = number of breakdowns in a week

$X \sim P_0(1.5)$

a $P(X \le 2) = 0.8088$ (tables)

b Let Y = no of breakdowns in a 2-week period

$Y \sim P_0(3)$

$$\begin{aligned} P(Y \ge 5) &= 1 - P(Y \le 4) \\ &= 1 - 0.8153 \quad \text{(tables)} \\ &= 0.1847 \end{aligned}$$

c Let B = number of breakdowns in a 6-week period

$B \sim P_0(9)$

Firm requires $P(B > n) \le 0.05$

i.e. $P(B \le n) \ge 0.95$

$P(B \le 13) = 0.9261 < 0.95$ (tables)

$P(B \le 14) = 0.9585 > 0.95$

$\therefore n = 14$

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Exercise D, Question 1

Question:

The random variable $X \sim B(80, 0.10)$. Using a suitable approximation, find

a $P(X \geq 1)$,

b $P(X \leq 6)$.

Solution:

$X \sim B(80, 0.10)$

$X \approx\sim P_0(8)$

a

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &\approx 1 - 0.0003 \qquad \text{(Poisson tables)} \\ &= 0.9997 \end{aligned}$$

b $P(X \leq 6) \approx 0.3134$ (Poisson tables)

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Exercise D, Question 2

Question:

The random variable $X \sim \mathrm{B}(120, 0.02)$. Using a suitable approximation, find

a $\mathrm{P}(X = 1)$,

b $\mathrm{P}(X \geq 3)$.

Solution:

$X \sim \mathrm{B}(120, 0.02)$

$X \approx\sim \mathrm{P_o}(2.4)$

a
$$\begin{aligned} \mathrm{P}(X=1) &= e^{-2.4}(2.4)^1 \\ &= 0.21772\ldots = 0.218\ (3\text{ s.f.}) \end{aligned}$$

b
$$\begin{aligned} \mathrm{P}(X \geq 3) &= 1 - \mathrm{P}(X \leq 2) \\ &= 1 - e^{-2.4}\left[1 + 2.4 + \frac{2.4^2}{2!}\right] \\ &= 1 - 0.56970\ldots \\ &= 0.430(3\text{ s.f.}) \end{aligned}$$

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Exercise D, Question 3

Question:

The random variable $X \sim B(50, 0.05)$. Find the percentage error in $P(X \le 4)$ when X is approximated by a Poisson distribution.

Solution:

$X \sim B(50, 0.05)$

$P(X \le 4) = 0.8964$ (Binomial tables)

$X \approx\sim P_o(2.5)$

$P(X \le 4) \approx 0.8912$ (Poisson tables)

$$\text{Percentage error} = \frac{(0.8964 - 0.8912)}{0.8964} \times 100$$

$$= 0.58\%$$

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Exercise D, Question 4

Question:

In a certain manufacturing process the proportion of defective articles produced is 2%. In a batch of 300 articles, use a suitable approximation to find the probability that

a there are fewer than 2 defectives,

b there are exactly 4 defectives.

Solution:

$X =$ number of defectives in a batch of 300

$X \sim \mathrm{B}(300, 0.02)$

$X \approx\sim \mathrm{P_0}(6)$

a $\mathrm{P}(X < 2) = \mathrm{P}(X \leq 1)$ (Poisson tables)

≈ 0.0174

b $\mathrm{P}(X = 4) = \mathrm{P}(X \leq 4) - \mathrm{P}(X \leq 3)$

$\approx 0.2851 - 0.1512$ (Poisson tables)

$= 0.1339$

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Exercise D, Question 5

Question:

A medical practice screens a random sample of 250 of its patients for a certain condition which is present in 1.5% of the population. Use a suitable approximation to find the probability that they obtain

a no patients with the condition,

b at least two patients with the condition.

Solution:

X = number of people with the condition in sample of 250

$X \sim B(250, 0.015)$

$X \approx\sim P_0(3.75)$

Poisson is suitable approximation.

a $P(X = 0) \approx e^{-3.75} = 0.0235177\ldots$ (using Poisson)

So using Poisson approximation probability ≈ 0.0235 (3 s.f.)

b
$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &\approx 1 - e^{-3.75}[1 + 3.75] \\ &= 1 - 4.75 \times e^{-3.75} \\ &= 1 - 0.111709\ldots \\ &= 0.8883 \end{aligned}$$

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Exercise D, Question 6

Question:

An experiment involving 2 fair dice is carried out 180 times. The dice are placed in a container, shaken and the number of times a double six is obtained recorded. Use a suitable approximation to find the probability that a double six is obtained

a once,

b twice,

c at least three times.

Solution:

$X =$ number of double sixes in 180 throws

$X \sim \text{B}(180, \frac{1}{36})$

$X \approx\sim \text{P}_0(5)$

a $\text{P}(X = 1) \approx \dfrac{e^{-5} \times 5^1}{1!} = 0.03368$

$= 0.0337 \quad (3 \text{ s.f.})$

b $\text{P}(X = 2) \approx \dfrac{e^{-5} \times 5^2}{2!} = 0.084224\ldots$

$= 0.0842 \ (3 \text{ s.f.})$

c $\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$

$\approx 1 - 0.1247$ (Poisson tables)

$= 0.8753$

$= 0.875 \ (3 \text{ s.f.})$

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Exercise D, Question 7

Question:

It is claimed that 95% of the population in a certain village are right-handed. A random sample of 80 villagers is tested to see whether or not they are right-handed. Use a Poisson approximation to estimate the probability that the number who are right-handed is

a 80,

b 79,

c at least 78.

Solution:

$X =$ number of villagers who are left-handed

$X \sim B(80, 0.05)$

(Need P small to use Poisson approximation)

$X \approx P_0(4)$

a $P(80 \text{ are right-handed}) = P(X = 0) \approx e^{-4}$ (Poisson Tables)

$= 0.0183$

b $P(79 \text{ are right-handed}) = P(X = 1)$

$= P(X \leq 1) - P(X = 0)$

$\approx 0.0916 - 0.0183$ (Poisson Tables)

$= 0.0733$

c $P(\text{at least 78 are right-handed}) = P(X \leq 2)$

≈ 0.2381 (Poisson tables)

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Exercise D, Question 8

Question:

In a computer simulation 500 dots were fired at a target and the probability of a dot hitting the target was 0.98. Find the probability that

a all the dots hit the target,

b at least 495 hit the target.

Solution:

X = number of hits out of 500

$X \sim \text{B}(500, 0.98)$

Y = number of misses out of 500 ($\because p$ is small and n large)

$Y \sim \text{B}(500, 0.02)$

$Y \approx \text{P}_0(10)$

a

$$\begin{aligned} \text{P}(X = 500) &= \text{P}(Y = 0) \\ &\approx e^{-10} = 0.000045399 \\ &\approx 0.0000454 \text{ (3 s.f.)} \end{aligned}$$

b

$$\begin{aligned} \text{P}(X \geq 495) &= \text{P}(Y \leq 5) \\ &\approx 0.0671 \quad \text{(Poisson tables)} \end{aligned}$$

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Exercise D, Question 9

Question:

a State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

Independently for each call into the telephone exchange of a large organisation, there is a probability of 0.002 that the call will be connected to a wrong extension.

b Find, to 3 significant figures, the probability that, on a given day, exactly one of the first 5 incoming calls will be wrongly connected.

c Use a Poisson approximation to find, to 3 decimal places, the probability that, on a day when there are 1000 incoming calls, at least 3 of them are wrongly connected during that day. ***E***

Solution:

a For large n
and small p
$B(n, p) \approx P_0(np)$

b $X =$ number of wrongly connected calls in sample of 5
$X \sim B(5, 0.002)$

$$P(X = 1) = 5 \times (0.002)^1 (0.998)^4$$
$$= 0.00992$$

c $Y =$ number of wrongly connected calls in sample of 1000
$Y \sim B(1000, 0.002)$
So $Y \approx\sim P_0(2)$

$$P(Y \geq 3) = 1 - P(Y \leq 2)$$
$$\approx 1 - 0.6767 \quad \text{(Poisson Tables)}$$
$$= 0.3233$$

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Exercise E, Question 1

Question:

a State conditions under which the Poisson distribution is a suitable model to use in statistical work.

Flaws in a certain brand of tape occur at random and at a rate of 0.75 per 100 metres.
Assuming a Poisson distribution for the number of flaws in a 400 metre roll of tape,

b find the probability that there will be at least one flaw.

c Show that the probability that there will be at most 2 flaws is 0.423 (to 3 decimal places).

In a batch of 5 rolls, each of length 400 metres,

d find the probability that at least 2 rolls will contain fewer than 3 flaws.

Solution:

a If the outcomes occur:
1. singly
2. at a constant rate
3. independently and at random then a Poisson distribution can be suitable.

b Let $F =$ number of flows in 400 m of tape

$F \sim P_0(3)$

$$\begin{aligned} P(F \geq 1) &= 1 - P(F = 0) \quad \text{(tables)} \\ &= 1 - 0.0498 \\ &= 0.9502 \end{aligned}$$

c

$$\begin{aligned} P(F \leq 2) &= 0.4232 \quad \text{(tables)} \\ &= 0.423 \text{ (3 d.p.)} \end{aligned}$$

d Let $R =$ number of rolls (out of 5) with fewer than 3 flaws.

$P(\text{fewer than 3 flows}) = P(F < 3) = P(F \leq 2) = 0.423.$

$R \sim B(5, 0.423)$

$$\begin{aligned} P(R \geq 2) &= 1 - P(R \leq 1) \\ &= 1 - \left[5 \times (0.423)^1 \times (0.577)^4 + (0.577)^5\right] \\ &= 1 - 0.29838\ldots \\ &= 0.702 \end{aligned}$$

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Exercise E, Question 2

Question:

An archer fires arrows at a target and for each arrow, independently of all others, the probability that it hits the bull's eye is $\frac{1}{8}$.

a Given that the archer fires 5 arrows, find the probability that fewer than 2 arrows hit the bull's eye.

The archer fires 5 arrows, collects them and then fires all 5 again.

b Find the probability that on both occasions fewer than 2 hit the bull's eye.

The archer now fires 60 arrows at the target. Using a suitable approximation find

c the probability that fewer than 10 hit the bull's eye,

d the greatest value of m such that the probability that the archer hits the bull's eye with at least m arrows is greater than 0.5.

Solution:

$X =$ number of arrows that hit bull's eye

$X \sim B(5, \frac{1}{8})$

a
$$\begin{aligned} P(X < 2) &= P(X \leq 1) \\ &= 5 \times \left(\frac{1}{8}\right) \times \left(\frac{7}{8}\right)^4 + \left(\frac{7}{8}\right)^5 \\ &= 0.87927\ldots = 0.879 \text{ (3 s.f.)} \end{aligned}$$

b
$$\begin{aligned} [P(X < 2)]^2 &= (0.87927)^2 \\ &= 0.773 \text{ (3 s.f.)} \end{aligned}$$

$Y =$ number of arrows that hit bull's eye out of 60

c $Y \sim B(60, \frac{1}{8})$

$Y \sim P_0(7.5)$

$P(Y < 10) = P(Y \leq 9)$ (Poisson tables)

≈ 0.7764

d Require $P(Y \geq m) > 0.5$

i.e. $P(Y \leq m-1) < 0.5$

$$\left.\begin{aligned} P(Y \leq 6) &= 0.3782 < 0.5 \\ P(Y \leq 7) &= 0.5246 > 0.5 \end{aligned}\right\} \text{(Using Poisson tables)}$$

$\therefore m - 1 = 6$

so $m = 7$

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Exercise E, Question 3

Question:

In Joe's roadside café $\frac{2}{5}$ of the customers buy a cup of tea.

a Find the probability that at least 4 of the next 10 customers will buy a cup of tea.

Joe has calculated that, on a typical morning, customers arrive in the café at a rate of 0.5 per minute.

b Find the probability that at least 10 customers arrive in the next 15 minutes.

c Find the probability that exactly 10 customers arrive in the next 20 minutes.

d Find the probability that in the next 20 minutes exactly 10 customers arrive and at least 4 of them buy a cup of tea.

Solution:

$X =$ number of customers out of 10 who buy a cup of tea

$X \sim \text{B}(10, 0.4)$

a
$$\begin{aligned} \text{P}(X \geq 4) &= 1 - \text{P}(X \leq 3) \\ &= 1 - 0.3823 \qquad \text{(binomial tables)} \\ &= 0.6177 \end{aligned}$$

$C =$ number of customers who arrive in next 15 minutes

$C \sim \text{P}_0(7.5)$ ← $\lambda = 7.5 = 0.5 \times 15$

b
$$\begin{aligned} \text{P}(C \geq 10) &= 1 - \text{P}(C \leq 9) \\ &= 1 - 0.7764 \\ &= 0.2236 \end{aligned}$$

$T =$ number of customers who arrive in next 20 minutes

$T \sim \text{P}_0(10)$ ← $\lambda = 20 \times 0.5 = 10$

c
$$\begin{aligned} \text{P}(T = 10) &= \text{P}(T \leq 10) - \text{P}(T \leq 9) \\ &= 0.5830 - 0.4579 \\ &= 0.1251 \end{aligned}$$

d
$$\begin{aligned} \text{P}(T = 10) \times \text{P}(X \geq 4) &= 0.1251 \times 0.6177 \\ &= 0.07727\ldots \\ &= 0.0773 \end{aligned}$$

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Exercise E, Question 4

Question:

The number, X, of breakdowns per week of the lifts in a large block of flats has a Poisson distribution with mean 0.25. Find, to 3 decimal places, the probability that in a particular week

a there will be at least one breakdown,

b there will be at most 2 breakdowns.

c Show that the probability that during a 12-week period there will be no lift breakdowns is 0.050 (to 3 decimal places).

The residents in the flats have a maintenance contract with *Liftserve*. The contract is for a set of 20, 12-week periods. For every 12-week period with no breakdowns the residents pay *Liftserve* £500. If there is at least 1 breakdown in a 12-week period then *Liftserve* will mend the lift free of charge and the residents pay nothing for that period of 12 weeks.

d Find the probability that over the course of the contract the residents pay no more than £1000.

Solution:

$X \sim \mathrm{P_0}(0.25)$

a
$$\begin{aligned} \mathrm{P}(X \geq 1) &= 1 - \mathrm{P}(X = 0) \\ &= 1 - e^{-0.25} \\ &= 0.221199\ldots \quad = 0.221 \text{ (3 d.p.)} \end{aligned}$$

b
$$\begin{aligned} \mathrm{P}(X \leq 2) &= \mathrm{P}(X=0) + \mathrm{P}(X=1) + \mathrm{P}(X=2) \\ &= e^{-0.25}\left[1 + 0.25 + \frac{0.25^2}{2!}\right] \\ &= 0.99783\ldots \quad = 0.998 \text{ (3 d.p.)} \end{aligned}$$

c $T =$ number of breakdowns in 12 weeks

$T \sim \mathrm{P_0}(3)$ ← $\lambda = 3 = 12 \times 0.25$

$$\begin{aligned} \therefore \mathrm{P}(T = 0) &= e^{-3} = 0.04978\ldots \\ &= 0.050 \text{ (3 d.p.)} \end{aligned}$$

d $Y =$ number of 12-week periods with no breakdowns out of 20

$Y \sim \mathrm{B}(20, 0.050)$

Residents pay $\leq$ £1000 if $Y \leq 2$

$$\begin{aligned} \mathrm{P}(Y \leq 2) &= 0.9245 \quad \text{(binomial tables)} \\ &= 0.925 \quad \text{(3 d.p.)} \end{aligned}$$

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Exercise E, Question 5

Question:

Accidents occur in a school playground at the rate of 3 per year.

a Suggest a suitable model for the number of accidents in the playground next month.

b Using this model calculate the probability of 1 or more accidents in the playground next month.

Solution:

a $X =$ number of accidents in a month

$\lambda = 3 \times \frac{1}{12} = 0.25$

$X \sim P_0(0.25)$

b $P(X \geq 1) = 1 - P(X = 0)$

$= 1 - e^{-0.25}$

$= 0.221$ (3 s.f.)

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Exercise F, Question 1

Question:

During working hours an office switchboard receives telephone calls at random and at a rate of one call every 40 seconds.

a Find, to 3 decimal places, the probability that during a given one-minute period

i no call is received,

ii at least 2 calls are received.

b Find, to 3 decimal places, the probability that no call is received between 10:30 a.m. and 10:31 a.m. and that at least two calls are received between 10:31 a.m. and 10:32 a.m. ***E***

Solution:

$X =$ number of calls received during one minute

$X \sim \mathrm{P_0}(1.5)$ since $\dfrac{1}{\left(\frac{2}{3}\right)} 1.5$

i

$$\begin{aligned} \mathrm{P}(X=0) &= 0.2231 \quad \text{(tables)} \\ &= 0.223 \text{ (3 d.p.)} \end{aligned}$$

ii

$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1-\mathrm{P}(X \leq 1) \\ &= 1-0.5578 \quad \text{(tables)} \\ &= 0.4422 \\ &= 0.442 \text{ (3 d.p.)} \end{aligned}$$

b

$$\begin{aligned} \mathrm{P}(X=0) &\times \mathrm{P}(X \geq 2) \\ &= 0.2231 \times 0.4422 \\ &= 0.09865\ldots \\ &= 0.0987 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 2

Question:

State conditions under which the Poisson distribution is a suitable model to use in statistical work.

The number of typing errors per 1000 words made by a typist has a Poisson distribution with mean 2.5.

a Find, to 3 decimal places, the probability that in an essay of 4000 words there will be at least 12 typing errors.

The typist types 3 essays, each of length 4000 words.

b Find the probability that each contains at least 12 typing errors. ***E***

Solution:

Items occur in continuous space or time:

1 singly

2 at a constant rate

3 independently of one another and at random.

a X = number of errors in 1,000 words

$X \sim \mathrm{P_o}(10)$ ← $\lambda = 4 \times 2.5$

$$\begin{aligned} \mathrm{P}(X \geq 12) &= 1 - \mathrm{P}(X \leq 11) \\ &= 1 - 0.6968 \\ &= 0.3032 \end{aligned}$$

b Y = number of 4000 word essays with at least 12 errors

$Y \sim \mathrm{B}(3, 0.3032)$

$$\begin{aligned} \mathrm{P}(Y = 3) &= (0.3032)^3 \\ &= 0.027873\ldots \\ &= 0.0279 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 3

Question:

a State conditions under which the binomial distribution $B(n, p)$ may be approximated by a Poisson distribution and write down the mean of this Poisson distribution.

Samples of blood were taken from 250 children in a region of India. Of these children, 4 had blood type $A2B$.

b Write down an estimate of p, the proportion of children in this region having blood type $A2B$.

Consider a group of n children from this region and let X be the number having blood type $A2B$. Assuming that X is distributed $B(n, p)$ and that p has the value estimated above, calculate, to 3 decimal places, the probability that the number of children with blood type $A2B$ in a group of 6 children from this region will be

i zero,

ii more than 1.

c Use a Poisson approximation to calculate, to 4 decimal places, the probability that, in a group of 800 children from this region, there will be fewer than 3 children of blood type $A2B$.

Solution:

a $B(n, p)$ can be approximated to $P_0(np)$

If n is large and p is small

Then mean $= np$

b $\hat{p} = \frac{4}{250} = 0.016$

$X \sim B(6, 0.016)$

i

$$P(X = 0) = (0.984)^6 = 0.907759\ldots = 0.908 \text{ (3 d.p.)}$$

ii

$$\begin{aligned} P(X > 1) &= 1 - P(X \le 1) \\ &= 1 - [6 \times 0.016 \times (0.984)^5 + (0.984)^6] \\ &= 0.003679\ldots \\ &= 0.00370 \text{ (3 s.f.)} \end{aligned}$$

c $Y =$ number of children out of 800 with the blood group $A2B$

$Y \sim B(800, 0.016)$

$Y \approx\sim P_0(12.8)$

$$\begin{aligned} P(Y < 3) &= P(Y \le 2) \\ &= e^{-12.8}\left[1 + 12.8 + \frac{12.8^2}{2!}\right] \\ &= 0.00026426\ldots \\ &= 0.000264 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 4

Question:

Which of the following variables is best modelled by a Poisson distribution and which is best modelled by a binomial distribution?

a The number of hits by an arrow on a target, when 20 arrows are fired.

b The number of earth tremors that take place in a village over a given period of time.

c The number of particles emitted per minute by a radioactive isotope.

d The number of heads you get when tossing 2 coins 100 times.

e The number of accidents in a city in a year.

f The number of flying bomb hits in specified areas of London during World War 2.

Solution:

a	Binomial	a fixed number of arrows $(n = 20)$ need to assume that $p =$ probability of an arrow hitting is constant
b	Poisson	no fixed number of trials need to assume earth tremors occur at random with a constant rate
c	Poisson	no fixed number of particles need to assume particles emitted at a constant average rate
d	Binomial	$n = 200$ the number of tosses of coins $p = \frac{1}{2}$
e	Poisson	no fixed number need to assume accidents occur at a constant rate
f	Poisson	no fixed number need to assume flying bomb hits occur at a constant rate

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Exercise F, Question 5

Question:

Loaves of bread on a production line pass a monitoring point at a constant rate of 300 loaves per hour.

a Find how many loaves you would expect to pass the monitoring point in 2 minutes.

b Find the probability that no loaves pass the monitoring point in a given 1-minute period.

Solution:

X = number of loaves passing in 2 minutes

a $X = \frac{300}{60} \times 2 = 10 = \mathrm{E}(X)$

b Y = number of loaves passing in 1 minute

$Y \sim \mathrm{P_o}(5)$

$\mathrm{P}(Y = 0) = \mathrm{e}^{-5} = 0.0067$ (tables)

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Exercise F, Question 6

Question:

Accidents occur at a certain road junction at a rate of 3 per year.

a Suggest a suitable model for the number of accidents at this road junction in the next month.

b Show that, under this model, the probability of 2 or more accidents at this road junction in the next month is 0.0265 to 4 decimal places.

The local residents have applied for a crossing to be installed.

The planning committee agree to monitor the situation for the next 12 months. If there is at least one month with 2 or more accidents in it they will install a crossing.

c Find the probability that the crossing is installed. ***E***

Solution:

a $\lambda = \frac{3}{12} = 0.25$

$X =$ number of accidents in a month

$X \sim \mathrm{P_0}(0.25)$

b
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - [\mathrm{P}(X = 1) + \mathrm{P}(X = 0)] \\ &= 1 - \mathrm{e}^{-0.25}[0.25 + 1] \\ &= 1 - 0.97350\ldots \\ &= 0.026499\ldots \\ &= 0.0265 \end{aligned}$$

$Y =$ number of months with 2 or more accidents

$Y \sim \mathrm{B}(12, 0.0265)$

c
$$\begin{aligned} \mathrm{P}(Y \geq 1) &= 1 - \mathrm{P}(Y = 0) \\ &= 1 - (0.9735)^{12} \\ &= 1 - 0.724488\ldots \\ &= 0.275511\ldots \\ &= 0.276 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 7

Question:

Breakdowns occur on a particular machine at a rate of 2.5 per month. Assuming that the number of breakdowns can be modelled by a Poisson distribution, find the probability that

a exactly 3 occur in a particular month,

b more than 10 occur in a three-month period,

c exactly 3 occur in each of 2 successive months. ***E***

Solution:

$X =$ number of breakdowns per month

$X \sim P_0(2.5)$

a $P(X = 3) = P(X \leq 3) - P(X \leq 2)$ (tables)

$= 0.7576 - 0.5438$

$= 0.2138$

$Y =$ number of breakdowns in 3 months

$Y \sim P_0(7.5)$

b $P(Y > 10) = 1 - P(Y \leq 10)$ (tables)

$= 1 - 0.8622$

$= 0.1378$

c $P(X = 3) \times P(X = 3)$

$= (0.2138)^2$

$= 0.04571$

$= 0.0457$

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Exercise F, Question 8

Question:

A geography student is studying the distribution of telephone boxes in a large rural area where there is an average of 300 boxes per 500 km^2. A map of part of the area is divided into 50 squares, each of area 1 km^2 and the student wishes to model the number of telephone boxes per square.

a Suggest a suitable model the student could use and specify any parameters required.

One of the squares is picked at random.

b Find the probability that this square does not contain any telephone boxes.

c Find the probability that this square contains at least 3 telephone boxes.

The student suggests using this model on another map of a large city and surrounding villages.

d Comment, giving your reason briefly, on the suitability of the model in this situation. ***E***

Solution:

X = number of telephone boxes per square

a $X \sim \mathrm{P_o}(0.6)$ ← $\lambda = \frac{300}{500} = 0.6$

b $\mathrm{P}(X = 0) = e^{-0.6} = 0.5488\ldots$

$= 0.549$ (3 s.f.)

c $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$

$= 1 - e^{-0.6}\left[1 + 0.6 + \frac{0.6^2}{2}\right]$

$= 1 - 0.97688$

$= 0.02312$

$= 0.0231$ (3 s.f.)

d Not suitable

The rate of telephone boxes will be different in cities and they are more likely to occur in clusters.

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Exercise F, Question 9

Question:

All the letters in a particular office are typed either by Pat, a trainee typist, or by Lyn, who is a fully-trained typist. The probability that a letter typed by Pat will contain one or more errors is 0.3.

a Find the probability that a random sample of 4 letters typed by Pat will include exactly one letter free from error.

The probability that a letter typed by Lyn will contain one or more errors is 0.05.

b Use tables, or otherwise, to find, to 3 decimal places, the probability that in a random sample of 20 letters typed by Lyn, not more than 2 letters will contain one or more errors.

On any one day, 6% of the letters typed in the office are typed by Pat. One letter is chosen at random from those typed on that day.

c Show that the probability that it will contain one or more errors is 0.065.

Given that each of 2 letters chosen at random from the day's typing contains one or more errors,

d find, to 4 decimal places, the probability that one was typed by Pat and the other by Lyn. ***E***

Solution:

$X =$ number of letters out of 4 with at least one error

$X \sim \text{B}(4, 0.3)$

a $\text{P}(X = 3) = 4 \times 0.3^3 \times 0.7^1 = 0.0756$

$L =$ number of letters out of 20 containing one or more errors

$L \sim \text{B}(20, 0.05)$

b $\text{P}(L \leq 2) = 0.9245$ (tables)

$= 0.925$ (3 d.p.)

c

Pat Lyn

$\text{P(letter has errors)} = 0.06 \times 0.3 + 0.94 \times 0.05 = 0.065$

d

Pat Lyn

$$\text{Probability} = \frac{2 \times (0.06 \times 0.3) \times (0.94 \times 0.05)}{(0.065)^2} = 0.4004733\ldots = 0.4005 \text{ (4 d.p.)}$$

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Exercise F, Question 10

Question:

The number of breakdowns per day of the lifts in a large block of flats is modelled by a Poisson distribution with mean 0.2.

a Find, to 3 decimal places, the probability that on a particular day there will be at least one breakdown.

b Find the probability that there are fewer than 2 days in a 30-day month with at least one breakdown.

Solution:

$X =$ number of breakdowns per day
$X \sim \mathrm{P_0}(0.2)$

a
$$\begin{aligned} \mathrm{P}(X \geq 1) &= 1 - \mathrm{P}(X = 0) \\ &= 1 - e^{-0.2} \\ &= 1 - 0.8187\ldots \\ &= 0.181 (3\text{ d.p.}) \end{aligned}$$

$Y =$ number of days in 30 day months with at least one breakdown
$Y \sim \mathrm{B}(30, 0.181)$

b
$$\begin{aligned} \mathrm{P}(Y < 2) &= \mathrm{P}(Y \leq 1) \\ &= (0.819)^{30} + 30(0.819)^{29} \times (0.181) \\ &= 0.0191 \end{aligned}$$

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Exercise A, Question 1

Question:

Give reasons why the following are not valid probability density functions.

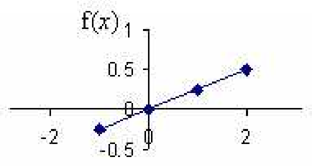
a $f(x) = \begin{cases} \frac{1}{4}x, & -1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$

b $f(x) = \begin{cases} x^2, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

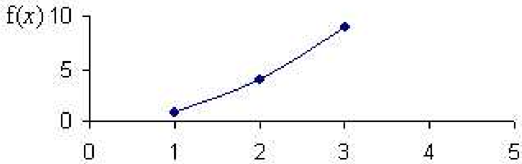
c $f(x) = \begin{cases} (x^3 - 2), & -1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a There are negative values for f(x) when $x < 0$ so this is not a probability density function.



b



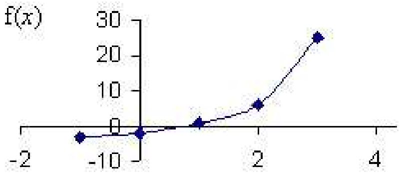
No negative values of f(x)

Area $= \int_1^3 x^2 \, dx$

$= \left[\frac{x^3}{3} \right]_1^3$

$= 8\frac{2}{3}$ not equal to 1 therefore it is not a valid probability density function.

c There are negative values for f(x) so this is not a probability density function.



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Exercise A, Question 2

Question:

For what value of k is the following a valid probability density function?

$$f(x) = \begin{cases} k(1-x^2), & -4 \le x \le -2, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

$$\int_{-4}^{-2} k - kx^2 \, dx = 1$$

$$\left[kx - \frac{kx^3}{3}\right]_{-4}^{-2} = 1$$

$$\left[-2k + \frac{8k}{3}\right] - \left[-4k + \frac{64k}{3}\right] = 1$$

$$-\frac{50}{3}k = 1$$

$$k = -\frac{3}{50}$$

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Exercise A, Question 3

Question:

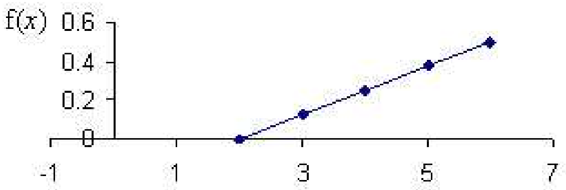
Sketch the following probability density functions.

a $f(x)=\begin{cases}\frac{1}{8}(x-2), & 2\le x\le 6,\\ 0, & \text{otherwise.}\end{cases}$

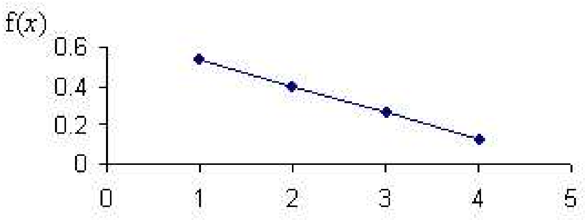
b $f(x)=\begin{cases}\frac{2}{15}(5-x), & 1\le x\le 4,\\ 0, & \text{otherwise.}\end{cases}$

Solution:

a



b



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Exercise A, Question 4

Question:

Find the value of k so that each of the following are valid probability density functions.

a $f(x) = \begin{cases} kx, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

b $f(x) = \begin{cases} kx^2, & 0 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

c $f(x) = \begin{cases} k(1+x^2), & -1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a

$$\int_1^3 kx \, dx = 1$$

$$\left[\frac{kx^2}{2}\right]_1^3 = 1$$

$$\frac{9k}{2} - \frac{k}{2} = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$\int_0^3 kx^2 dx = 1$$

$$\left[\frac{kx^3}{3}\right]_0^3 = 1$$

$$\frac{27k}{3} = 1$$

$$27k = 3$$

$$k = \frac{3}{27} = \frac{1}{9}$$

c

$$\int_{-1}^2 k(1+x^2) \, dx = 1$$

$$\left[kx + \frac{kx^3}{3}\right]_{-1}^2 = 1$$

$$(2k + \frac{8k}{3}) - (-k - \frac{k}{3}) = 1$$

$$\frac{14k}{3} - (-\frac{4k}{3}) = 1$$

$$\frac{14k}{3} + \frac{4k}{3} = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

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Exercise A, Question 5

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4-x), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the value of k.

b Sketch the probability density function for all values of x.

Solution:

a

$$\int_0^2 k(4-x)\,dx = 1$$

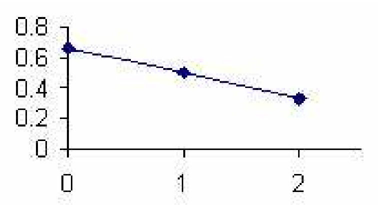
$$\left[4kx - \frac{kx^2}{2}\right]_0^2 = 1$$

$$8k - 2k = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

b



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Exercise A, Question 6

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^2(2-x), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k.

Solution:

$$\int_0^2 kx^2(2-x)\,dx = 1$$

$$\left[\frac{2kx^3}{3} - \frac{kx^4}{4}\right]_0^2 = 1$$

$$\left(\frac{16k}{3} - \frac{16k}{4}\right) - 0 = 1$$

$$\frac{16k}{12} = 1$$

$$16k = 12$$

$$k = \frac{3}{4} \text{ or } 0.75$$

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Exercise A, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^3, & 1 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k.

Solution:

$$\int_1^4 kx^3 \, dx = 1$$

$$\left[\frac{kx^4}{4} \right]_1^4 = 1$$

$$\frac{256k}{4} - \frac{k}{4} = 1$$

$$\frac{255k}{4} = 1$$

$$k = \frac{4}{255}$$

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Exercise A, Question 8

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & 0 < x < 2, \\ k(2x-3), & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the value of k.

b Sketch the probability density function for all values of x.

Solution:

a

$$\int_0^2 k \, dx + \int_2^3 k(2x-3) \, dx = 1$$

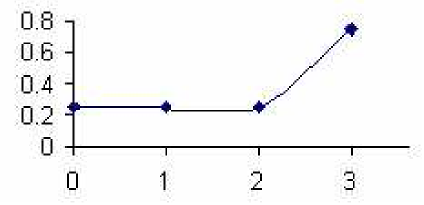
$$[kx]_0^2 + \left[\frac{2kx^2}{2} - 3kx\right]_2^3 = 1$$

$$2k + [(9k - 9k) - (4k - 6k)] = 1$$

$$2k + 2k = 1$$

$$k = \frac{1}{4} \text{ or } 0.25$$

b



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Exercise B, Question 1

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{3x^2}{8}, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

1 Method 1: $F(x) = \int_0^x \frac{3t^2}{8}\,dt$ $= \left[\frac{3t^3}{24}\right]_0^x$ $= \frac{3x^3}{24} - 0$ $= \frac{3x^3}{24}$	**Method 2:** $F(x) = \int \frac{3x^2}{8}\,dx$ $= \frac{3x^3}{24} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x^3}{24} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

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Exercise B, Question 2

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{1}{4}(4-x), & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

2 Method 1: $F(x) = \int_1^x \frac{1}{4}(4-t)\,dt$ $= \left[t - \frac{t^2}{8}\right]_1^x$ $= \left(x - \frac{x^2}{8}\right) - \left(1 - \frac{1}{8}\right)$ $= x - \frac{x^2}{8} - \frac{7}{8}$	**Method 2:** $F(x) = \int \frac{1}{4}(4-x)\,dx$ $= x - \frac{x^2}{8} + C$ $F(3) = 1$ $3 - \frac{9}{8} + C = 1$ $C = -\frac{7}{8}$
$F(x) = \begin{cases} 0 & x < 1 \\ x - \frac{x^2}{8} - \frac{7}{8} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$	

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Exercise B, Question 3

Question:

The continuous random variable X has probability density function given by:

$$\mathrm{F}(x)=\begin{cases}\frac{x}{9}, & 0<x<3,\\ \frac{1}{9}(6-x) & 3\le x\le 6,\\ 0, & \text{otherwise.}\end{cases}$$

Find $\mathrm{F}(x)$.

Solution:

Method 1:	Method 2:
$\mathrm{F}(x)=\int_0^x \frac{t}{9}\,\mathrm{d}t$ $=\left[\frac{t^2}{18}\right]_0^x$ $=\frac{x^2}{18}$ $\mathrm{F}(x)=\int_3^x \frac{1}{9}(6-t)\,\mathrm{d}t+\int_0^3 \frac{x}{9}\,\mathrm{d}x$ $=\left[\frac{2t}{3}-\frac{t^2}{18}\right]_3^x+\left[\frac{x^2}{18}\right]_0^3$ $=\left(\frac{2x}{3}-\frac{x^2}{18}\right)-\left(2-\frac{9}{18}\right)+0.5$ $=\frac{2x}{3}-\frac{x^2}{18}-1$	$\mathrm{F}(x)=\int \frac{x}{9}\,\mathrm{d}x$ $=\frac{x^2}{18}+C$ $\mathrm{F}(0)=0$ $0+C=0$ $C=0$ $\mathrm{F}(x)=\int \frac{1}{9}(6-x)\,\mathrm{d}x$ $=\frac{2x}{3}-\frac{x^2}{18}+C$ $\mathrm{F}(6)=1$ $4-2+C=1$ $C=-1$
$F(x)=\begin{cases}0 & x\le 0\\ \frac{x^2}{18} & 0<x<3\\ \frac{2x}{3}-\frac{x^2}{18}-1 & 3\le x\le 6\\ 1 & x>6\end{cases}$	

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Exercise B, Question 4

Question:

The continuous random variable X has probability density function given by:

$$f(x)=\begin{cases} k, & 0 \le x \le 3, \\ k(2x-5), & 3 \le x \le 5 \\ 0, & \text{otherwise.} \end{cases}$$

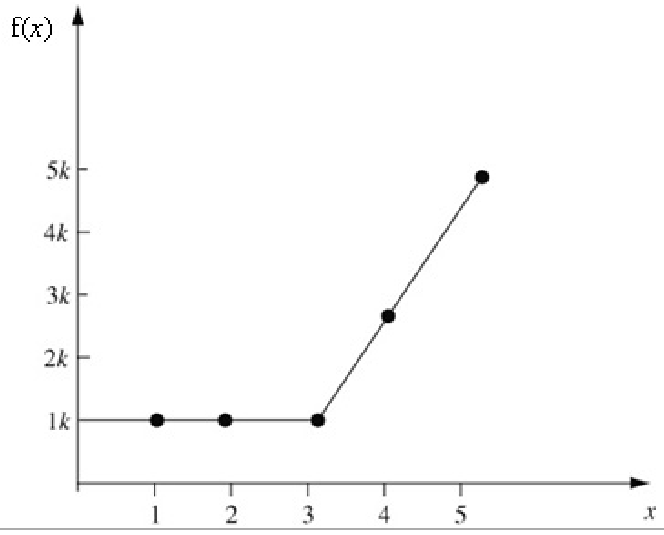
a Sketch $f(x)$.

b Find the value of k.

c Find $F(x)$.

Solution:

a



b

$$\int_0^3 k\,dx + \int_3^5 k(2x-5)\,dx = 1$$

$$[kx]_0^3 + \left[k(x^2-5x)\right]_3^5 = 1$$

$$3k + \left[k(25-25) - k(9-15)\right] = 1$$

$$9k = 1$$

$$k = \frac{1}{9}$$

c Method 1

$$\int_0^x \frac{1}{9}\,dt = \left[\frac{1}{9}t\right]_0^x$$

$$= \frac{1}{9}x$$

$$\int_3^x \frac{1}{9}(2t-5)\,dt + \int_0^3 \frac{1}{9}\,dx = \left[\frac{1}{9}(t^2-5t)\right]_3^x + \left[\frac{x}{9}\right]_0^3$$

$$= \left[\frac{1}{9}(x^2-5x) - \frac{1}{9}(9-15)\right] + \left[\frac{3}{9}\right]$$

$$= \frac{1}{9}x^2 - \frac{5}{9}x + 1$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{9} & 0 < x < 3 \\ \frac{x^2}{9} - \frac{5x}{9} + 1 & 3 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

Method 2

$$\int \frac{1}{9}\,dx = \frac{1}{9}x + C$$

$$F(0) = 0$$

$$C = 0$$

$$\int \frac{1}{9}(2x-5)\,dx = \frac{1}{9}(x^2-5x) + C$$

$$F(5) = 1$$

$$\frac{1}{9}(25-25) + C = 1$$

$$C = 1$$

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Exercise B, Question 5

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

Find the probability density function, f(x).

Solution:

$\frac{d}{dx}F(x) = \frac{2x}{5}$ $f(x) = \begin{cases} \frac{2x}{5} & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$	

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Exercise B, Question 6

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{2}(x-1), & 1 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

a Find $P(X \le 2.5)$.

b Find $P(X > 1.5)$.

c Find $P(1.5 \le X \le 2.5)$.

Solution:

a $P(X \le 2.5) = F(2.5)$ $= \frac{1}{2}(2.5-1)$ $= 0.75$ **b** $P(X > 1.5) = 1 - F(1.5)$ $= 1 - \frac{1}{2}(1.5-1)$ $= 0.75$ **c** $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5)$ $= 0.75 - 0.25$ $= 0.5$	

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Exercise B, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{3x^2}{8} & 0 \le x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the cumulative distribution function of X.

b Find $P(X \le 1)$.

Solution:

a Method 1: $F(X) = \int_0^x \frac{3x^2}{8}\,dx$ $= \left[\frac{x^3}{8}\right]_0^x$ $= \frac{x^3}{8}$	**Method 2:** $F(X) = \int \frac{3x^2}{8}\,dx$ $= \frac{x^3}{8} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$	
b $P(x \le 1) = F(1) = \frac{1}{8}$	

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Exercise B, Question 8

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x)=\begin{cases}0, & x<1,\\ \frac{1}{2}(x^3-2x^2+x), & 1\le x\le 2,\\ 1, & x>2.\end{cases}$$

a Find the probability density function $f(x)$.
b Sketch the probability density function.
c Find $P(X<1.5)$.

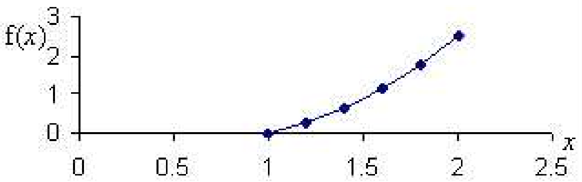
Solution:

a

$$\frac{d}{dx}F(x)=\frac{3x^2}{2}-2x+\frac{1}{2}$$

$$f(x)=\begin{cases}\frac{3}{2}x^2-2x+\frac{1}{2} & 1\le x\le 2\\ 0 & \text{otherwise}\end{cases}$$

b



c

$$\begin{aligned}P(X<1.5)&=F(1.5)\\ &=\tfrac{1}{2}(1.5^3-2\times 1.5^2+1.5)\\ &=0.1875\end{aligned}$$

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Exercise B, Question 9

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4 - x^2), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Show that $k = \frac{3}{16}$.

b Find the cumulative distribution function of X.

c Find $P(0.69 < X < 0.70)$. Give your answer correct to one significant figure.

Solution:

a $\int_0^2 k(4 - x^2)\,dx = 1$ $\left[k\left(4x - \frac{x^3}{3}\right)\right]_0^2 = 1$ $k\left(8 - \frac{8}{3}\right) = 1$ $\frac{16k}{3} = 1$ $k = \frac{3}{16}$	
b **Method 1**: $F(x) = \int_0^x \frac{3}{16}(4 - t^2)\,dt$ $= \left[\frac{3}{16}\left(4t - \frac{t^3}{3}\right)\right]_0^x$ $= \frac{3}{16}\left(4x - \frac{x^3}{3}\right)$ $F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{16}\left(4x - \frac{x^3}{3}\right) & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$ **c** $P(0.69 < X < 0.70) = F(0.70) - F(0.69)$ $= \frac{3}{16}\left(2.8 - \frac{0.343}{3}\right) - \frac{3}{16}\left(2.76 - \frac{0.328509}{3}\right)$ $= 0.00659$ $= 0.007$ (1 s.f.)	**Method 2**: $F(x) = \int \frac{3}{16}(4 - x^2)\,dx$ $= \frac{3}{16}\left(4x - \frac{x^3}{3}\right) + C$ $F(2) = 1$ $\frac{3}{16}\left(8 - \frac{8}{3}\right) + C = 1$ $C = 0$

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Exercise C, Question 1

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} kx^2, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a k,

b $E(X)$,

c $Var(X)$.

Solution:

a

$$\int_0^2 kx^2 = 1$$

$$\left[\frac{kx^3}{3}\right]_0^2 = 1$$

$$\frac{8k}{3} - 0 = 1$$

$$8k = 3$$

$$k = \frac{3}{8}$$

b

$$E(X) = \int_0^2 \frac{3x^3}{8}\, dx$$

$$= \left[\frac{3x^4}{32}\right]_0^2$$

$$= \frac{48}{32} - 0$$

$$= 1.5$$

c

$$Var(X) = \int_0^2 \frac{3x^4}{8}\, dx - 1.5^2$$

$$= \left[\frac{3x^5}{40}\right]_0^2 - 1.5^2$$

$$= \left(\frac{96}{40} - 0\right) - 2.25$$

$$= 2.4 - 2.25$$

$$= 0.15$$

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Exercise C, Question 2

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y^2}{9}, & 0 \le y \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $E(Y)$.

b Find $Var(Y)$.

c Find the standard deviation of Y.

Solution:

a

$$E(Y) = \int_0^3 \frac{y^3}{9}\,dy$$
$$= \left[\frac{y^4}{36}\right]_0^3$$
$$= \frac{81}{36} - 0$$
$$= 2.25$$

b

$$Var(Y) = \int_0^3 \frac{y^4}{9}\,dy - 2.25^2$$
$$= \left[\frac{y^5}{45}\right]_0^3 - 2.25^2$$
$$= \left(\frac{243}{45} - 0\right) - 5.0625$$
$$= 5.4 - 5.0625$$
$$= 0.3375$$

c $\sigma = \sqrt{0.3375} = 0.581$

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Exercise C, Question 3

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y}{8}, & 0 \le y \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $E(Y)$.

b Find $Var(Y)$.

c Find the standard deviation of Y.

d Find $P(Y > \mu)$.

e Find $Var(3Y + 2)$.

f Find $E(Y + 2)$.

Solution:

a

$$\mathrm{E}(Y) = \int_0^4 \frac{y^2}{8}\,dy$$
$$= \left[\frac{y^3}{24}\right]_0^4$$
$$= \frac{64}{24} - 0$$
$$= \frac{8}{3}$$

b

$$\mathrm{Var}(Y) = \int_0^4 \frac{y^3}{8}\,dy - \left(\frac{8}{3}\right)^2$$
$$= \left[\frac{y^4}{32}\right]_0^4 - \left(\frac{64}{9}\right)$$
$$= \left(\frac{256}{32} - 0\right) - \left(\frac{64}{9}\right)$$
$$= 8 - \left(\frac{64}{9}\right)$$
$$= \frac{8}{9}$$

c $\sigma = \sqrt{\frac{8}{9}} = 0.943$

d

$$\mathrm{P}(Y > \mu) = \mathrm{P}(Y > \tfrac{8}{3})$$
$$= \int_{\frac{8}{3}}^4 \frac{y}{8}\,dy$$
$$= \left[\frac{y^2}{16}\right]_{\frac{8}{3}}^4$$
$$= 1 - 0.4444$$
$$= 0.556$$

e

$$\mathrm{Var}\,(3Y + 2) = 9\ \mathrm{Var}(Y)$$
$$= 9 \times \frac{8}{9}$$
$$= 8$$

f

$$\mathrm{E}\,(Y + 2) = \mathrm{E}(Y) + 2$$
$$= \frac{8}{3} + 2$$
$$= 4\frac{2}{3}$$

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Exercise C, Question 4

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Find k.

b Find $E(X)$.

c Show that $\text{Var}(X) = \frac{1}{18}$.

d Find $P(X > \mu)$.

Solution:

a

$$\int_0^1 k(1-x)\,dx = 1$$

$$\left[kx - \frac{kx^2}{2}\right]_0^1 = 1$$

$$k - \tfrac{1}{2}k = 1$$

$$k = 2$$

b

$$E(X) = \int_0^1 (2x - 2x^2)\,dx$$

$$= \left[\frac{2x^2}{2} - \frac{2x^3}{3}\right]_0^1$$

$$= 1 - \tfrac{2}{3}$$

$$= \tfrac{1}{3}$$

c

$$\text{Var}(X) = \int_0^1 (2x^2 - 2x^3)\,dx \; - \left(\tfrac{1}{3}\right)^2 dx$$

$$= \left[\frac{2x^3}{3} - \frac{2x^4}{4}\right]_0^1 - \tfrac{1}{9}$$

$$= \left(\tfrac{2}{3} - \tfrac{1}{2}\right) - 0 - \tfrac{1}{9}$$

$$= \tfrac{1}{18}$$

d

$$P(X > \tfrac{1}{3}) = \int_{\frac{1}{3}}^1 2(1 - x)\,dx$$

$$= \left[2x - x^2\right]_{\frac{1}{3}}^1$$

$$= (2 - 1) - \left(\tfrac{2}{3} - \tfrac{1}{9}\right)$$

$$= \tfrac{4}{9}$$

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Exercise C, Question 5

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $P(X < 0.5)$.

b Find $E(X)$.

Solution:

a

$$\begin{aligned} P(X < 0.5) &= \int_0^{0.5} 12x^2 - 12x^3 \ dx \\ &= \left[4x^3 - 3x^4\right]_0^{0.5} \\ &= \frac{1}{2} - \frac{3}{16} \\ &= \frac{5}{16} \text{ or } 0.3125 \end{aligned}$$

b

$$\begin{aligned} E(X) &= \int_0^1 12x^3 - 12x^4 \ dx \\ &= \left[3x^4 - \frac{12x^5}{5}\right]_0^1 \\ &= (3 - 2.4) - 0 \\ &= 0.6 \text{ or } \frac{3}{5} \end{aligned}$$

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Exercise C, Question 6

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{3}{8}(1+x^2), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

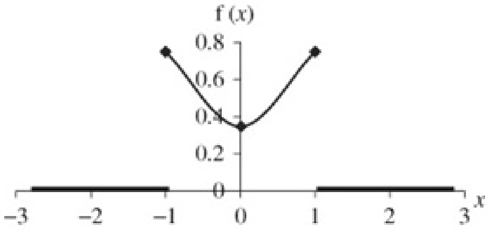
b Write down E(X).

c Show that $\sigma^2 = 0.4$.

d Find $P(-\sigma < X < \sigma)$.

Solution:

a



b

$E(X) = 0$ (by symmetry)

c

$$\sigma^2 = \text{Var}(X)$$
$$= \int_{-1}^{1} \frac{3x^2}{8} + \frac{3x^4}{8}\, dx - 0^2$$
$$= \left[\frac{3x^3}{24} + \frac{3x^5}{40}\right]_{-1}^{1}$$
$$= \left(\frac{3}{24} + \frac{3}{40}\right) - \left(-\frac{3}{24} - \frac{3}{40}\right)$$
$$= 0.4$$

d

$$P(-\sqrt{0.4} < X < \sqrt{0.4}) = \int_{-\sqrt{0.4}}^{\sqrt{0.4}} \frac{3}{8} + \frac{3x^2}{8}\, dx$$
$$= \left[\frac{3x}{8} + \frac{3x^3}{24}\right]_{-\sqrt{0.4}}^{\sqrt{0.4}}$$
$$= \left(\frac{3}{8} \times \sqrt{0.4} + \frac{3}{24} \times \left(\sqrt{0.4}\right)^3\right) - \left(\frac{3}{8} \times \left(-\sqrt{0.4}\right) + \frac{3}{24} \times \left(-\sqrt{0.4}\right)^3\right)$$
$$= 0.538$$

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Exercise C, Question 7

Question:

The continuous random variable T has c.d.f. given by

$$f(t) = \begin{cases} kt^3, & 0 \leq t \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

where k is a positive constant.

a Find k.

b Show that $E(T)$ is 1.6.

c Find $E(2T+3)$.

d Find $Var(T)$.

e Find $Var(2T+3)$.

f Find $P(T < 1)$.

Solution:

a

$$\int_0^2 kt^3 \, dt = 1$$

$$\left[\frac{kt^4}{4}\right]_0^2 = 1$$

$$4k - 0 = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$E(T) = \int_0^2 \frac{t^4}{4} \, dx$$

$$= \left[\frac{t^5}{20}\right]_0^2$$

$$= \frac{32}{20} - 0$$

$$= 1.6$$

c

$$E(2T + 3) = 2E(T) + 3$$

$$= 2 \times 1.6 + 3$$

$$= 6.2$$

d

$$Var(T) = \int_0^2 \frac{t^5}{4} \, dt - \left(\frac{8}{5}\right)^2$$

$$= \left[\frac{t^6}{24}\right]_0^2 - \left(\frac{8}{5}\right)^2$$

$$= \left(\frac{64}{24} - 0\right) - \left(\frac{64}{25}\right)$$

$$= \frac{8}{75}$$

e

$$Var(2T + 3) = 4 \, Var(T)$$

$$= \frac{32}{75}$$

f

$$P(T < 1) = \int_0^1 \frac{t^3}{4} \, dt$$

$$= \left[\frac{t^4}{16}\right]_0^1$$

$$= \frac{1}{16}$$

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Exercise C, Question 8

Question:

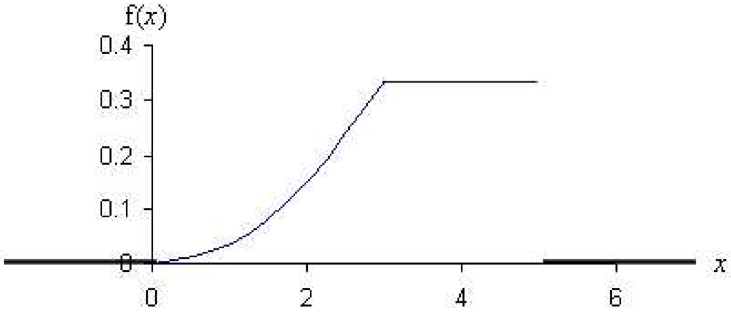
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{x^2}{27}, & 0 \le x < 3, \\ \frac{1}{3}, & 3 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

a Draw a rough sketch of $f(x)$.
b Find $E(X)$.
c Find $Var(X)$
d Find the standard deviation, σ, of X.

Solution:

a



b

$$E(X) = \int_0^3 \frac{x^3}{27}\,dx + \int_3^5 \frac{x}{3}\,dx$$
$$= \left[\frac{x^4}{108}\right]_0^3 + \left[\frac{x^2}{6}\right]_3^5$$
$$= \left(\frac{81}{108} - 0\right) + \left(\frac{25}{6} - \frac{9}{6}\right)$$
$$= \frac{41}{12}$$
$$= 3.417$$

c

$$\text{Var}(X) = \left(\int_0^3 \frac{x^4}{27}\,dx + \int_3^5 \frac{x^2}{3}\,dx\right) - \left(\frac{41}{12}\right)^2$$
$$= \left(\left[\frac{x^5}{135}\right]_0^3 + \left[\frac{x^3}{9}\right]_3^5\right) - \left(\frac{1681}{144}\right)$$
$$= \left(\frac{243}{135} - 0\right) + \left(\frac{125}{9} - \frac{27}{9}\right) - \left(\frac{1681}{144}\right)$$
$$= 1.0152$$

d

$$\sigma = \sqrt{1.0152} = 1.01$$

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Exercise C, Question 9

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}(x-1), & 1 \le x < 2, \\ \frac{1}{6}(5-x), & 2 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

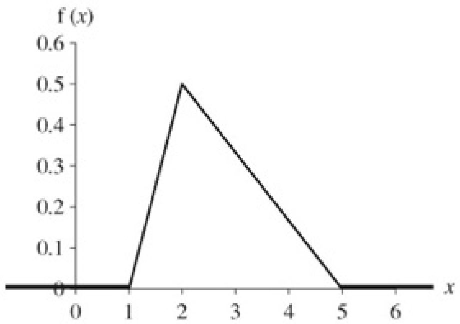
a Sketch $f(x)$.

b Find $E(X)$.

c Find $Var(X)$.

Solution:

a



b

$$E(X) = \int_1^2 \left(\frac{x^2}{2} - \frac{x}{2}\right) dx + \int_2^5 \left(\frac{5x}{6} - \frac{x^2}{6}\right) dx$$

$$= \left[\frac{x^3}{6} - \frac{x^2}{4}\right]_1^2 + \left[\frac{5x^2}{12} - \frac{x^3}{18}\right]_2^5$$

$$= \left[\left(\frac{8}{6} - 1\right) - \left(\frac{1}{6} - \frac{1}{4}\right)\right] + \left[\left(\frac{125}{12} - \frac{125}{18}\right) - \left(\frac{20}{12} - \frac{8}{18}\right)\right]$$

$$= 2\frac{2}{3}$$

c

$$Var(X) = \int_1^2 \left(\frac{x^3}{2} - \frac{x^2}{2}\right) dx + \int_2^5 \left(\frac{5x^2}{6} - \frac{x^3}{6}\right) dx - \left(2\frac{2}{3}\right)^2$$

$$= \left[\frac{x^4}{8} - \frac{x^3}{6}\right]_1^2 + \left[\frac{5x^3}{18} - \frac{x^4}{24}\right]_2^5 - \left(2\frac{2}{3}\right)^2$$

$$= \left[\left(\frac{16}{8} - \frac{8}{6}\right) - \left(\frac{1}{8} - \frac{1}{6}\right)\right] + \left[\left(\frac{625}{18} - \frac{625}{24}\right) - \left(\frac{40}{18} - \frac{16}{24}\right)\right] - \left(2\frac{2}{3}\right)^2$$

$$= \frac{13}{18}$$

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Exercise C, Question 10

Question:

Telephone calls arriving at a company are referred immediately by the telephonist to other people working in the company. The time a call takes, in minutes, is modelled by a continuous random variable T, having a p.d.f. given by

$$f(t) = \begin{cases} kt^2, & 0 \le t \le 10, \\ 0, & \text{otherwise.} \end{cases}$$

a Show that $k = 0.003$.

b Find $E(T)$.

c Find $Var(T)$.

d Find the probability of a call lasting between 7 and 9 minutes.

e Sketch the p.d.f.

Solution:

a

$$\int_0^{10} kt^2\,dt = 1$$

$$\left[\frac{kt^3}{3}\right]_0^{10} = 1$$

$$\frac{1000k}{3} - 0 = 1$$

$$1000k = 3$$

$$k = 0.003$$

b

$$E(T) = \int_0^{10} 0.003t^3\,dt$$

$$= \left[\frac{0.003t^4}{4}\right]_0^{10}$$

$$= \frac{30}{4} - 0$$

$$= 7.5$$

c

$$\text{Var}(x) = \int_0^{10} 0.003t^4\,dt - 7.5^2$$

$$= \left[\frac{0.003t^5}{5}\right]_0^{10} - 7.5^2$$

$$= (60 - 0) - 56.25$$

$$= 3.75$$

d

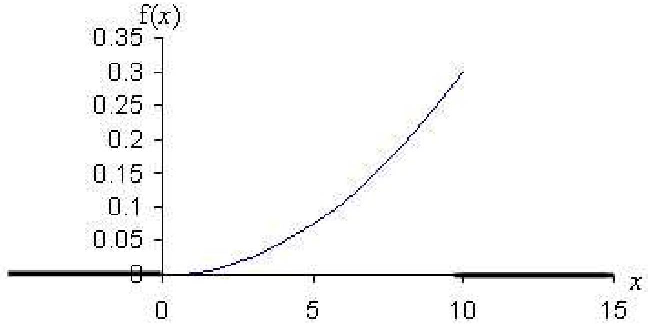
$$P(7 < T < 9) = \int_7^9 0.003t^2\,dt$$

$$= \left[\frac{0.003t^3}{3}\right]_7^9$$

$$= 0.729 - 0.343$$

$$= 0.386$$

e



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Exercise D, Question 1

Question:

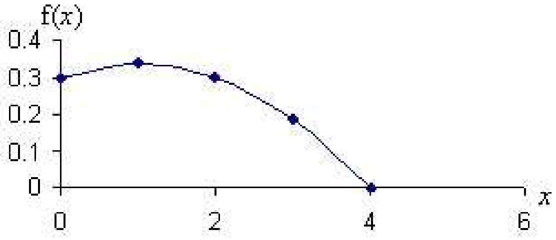
The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{80}(8+2x-x^2), & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.
b Find the mode of X.

Solution:

a



b Differentiating $\frac{d}{dx}\frac{3}{80}(8+2x-x^2)=0$

$$\frac{3}{80}(2-2x)=0$$

This $=0$ when $(2-2x)=0$

$$x=1$$

The mode is 1.
(Note: To check this is a maximum you could differentiate again and see if f'' is negative for all values of x.)

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Exercise D, Question 2

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{8}x, & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

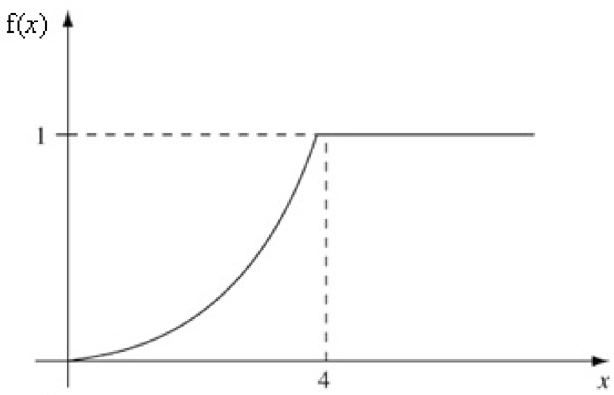
a Sketch the c.d.f. of X.

b Find the median of X.

Solution:

a

Method 1	**Method 2**
$\int_0^x \frac{1}{8}t \, dx = \left[\frac{t^2}{16}\right]_0^x$ $= \frac{x^2}{16}$	$F(x) = \int \frac{1}{8}x \, dx$ $= \frac{x^2}{16} + C$ $F(4) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \le x \le 4 \\ 1 & x > 4 \end{cases}$	



b $\frac{m^2}{16} = 0.5$ so $m^2 = 8$

$m = \sqrt{8} = 2.83$ or -2.83

Median $= 2.83$ since -2.83 is not in the range.

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Exercise D, Question 3

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{6}, & 0 \le x \le 2, \\ -\frac{x^2}{3} + 2x - 2, & 2 \le x \le 3 \\ 1, & x \ge 3. \end{cases}$$

a Find the median value of X. Give your answer to 3 decimal places.

b Find the quartiles and the inter-quartile range of X. Give your answer to 3 decimal places.

Solution:

a $F(m) = 0.5$ where m is the median.

Since $F(2) = \frac{2}{3}$ the median must lie in the range $0 \le x \le 2$

So $F(m) = \frac{m^2}{6} = 0.5$

$m^2 = 2$

$m = 1.73$ or -1.73

Median $= 1.73$ since -1.73 is not in the range.

b Lower quartile lies in the range $0 \le x \le 2$

$\frac{Q_1^2}{6} = 0.25$

$Q_1 = \sqrt{1.5} = 1.225$

Upper quartile lies in the range $2 \le x \le 3$

$-\frac{Q_3^2}{3} + 2Q_3 - 2 = 0.75$

$-Q_3^2 + 6Q_3 - 6 = 2.25$

$-Q_3^2 + 6Q_3 - 8.25 = 0$

$Q_3 = \frac{-6 \pm \sqrt{36 - 33}}{-2}$

$= 2.134$ or 3.87

$Q_3 = 2.134$ as 3.87 does not lie in the range

IQR $= 2.134 - 1.225 = 0.909$

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Exercise D, Question 4

Question:

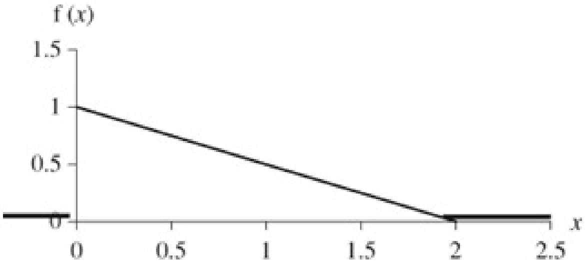
The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.
b Write down the mode of X.
c Find the c.d.f. of X.
d Find the median value of X.

Solution:

a



b 0

c

Method 1	Method 2
$\int_0^x \left(1 - \frac{1}{2}t\right) dt = \left[t - \frac{1}{4}t^2\right]_0^x$ $= x - \frac{1}{4}x^2$	$F(x) = \int 1 - \frac{1}{2}x \, dx$ $= x - \frac{1}{4}x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ x - \frac{1}{4}x^2 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

d $m - \frac{1}{4}m^2 = 0.5$

$m^2 - 4m + 2 = 0$

$$m = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$m = 2 - \sqrt{2}$ or $2 + \sqrt{2}$ therefore median $= 2 - \sqrt{2}$ as $2 + \sqrt{2}$ is not in range.

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Exercise D, Question 5

Question:

The continuous random variable Y has p.d.f. given by

$$f(x)=\begin{cases}\frac{1}{2}-\frac{1}{9}y, & 0\le y\le 3,\\ 0, & \text{otherwise.}\end{cases}$$

a Sketch the p.d.f. of Y.

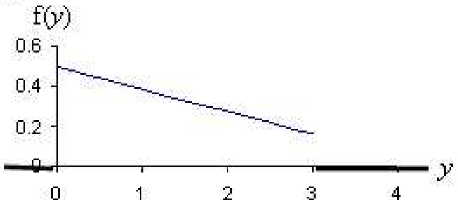
b Write down the mode of Y.

c Find the c.d.f. of Y.

d Find the median value of Y.

Solution:

a



b 0

c

Method 1	Method 2
$\int_0^y \frac{1}{2}-\frac{1}{9}t\ dt=\left[\frac{t}{2}-\frac{1}{18}t^2\right]_0^y$ $=\frac{y}{2}-\frac{1}{18}y^2$	$F(x)=\int \frac{y}{2}-\frac{1}{9}y\ dy$ $=\frac{y}{2}-\frac{1}{18}y^2+C$ $F(3)=1 \quad 1=\frac{3}{2}-\frac{9}{18}+C$ $C=0$
$F(y)=\begin{cases}0 & y<0\\ \frac{y}{2}-\frac{1}{18}y^2 & 0\le y\le 3\\ 1 & y>3\end{cases}$	

d $\frac{m}{2}-\frac{1}{18}m^2=0.5$

$m^2-9m+9=0$

$m=\frac{9\pm\sqrt{81-36}}{2}$

$\text{median}=\frac{9-3\sqrt{5}}{2}=1.15$

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Exercise D, Question 6

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{4}x^3, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

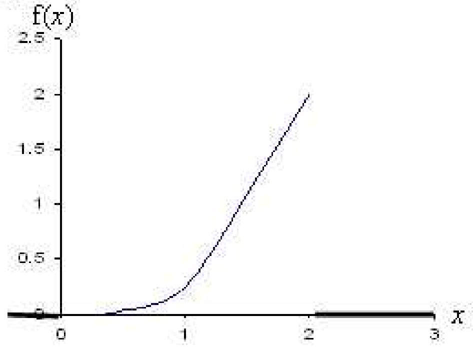
b Write down the mode of X.

c Find the c.d.f. of X.

d Find the median value of X.

Solution:

a



b 2

c

Method 1	Method 2
$\int_0^x \frac{1}{4}t^3 dt = \left[\frac{1}{16}t^4\right]_0^x = \frac{1}{16}x^4$	$F(x) = \int \frac{1}{4}x^3 dx = \frac{1}{16}x^4 + C$ $F(2) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16}x^4 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

d $\frac{1}{16}m^4 = 0.5$

$m^4 = 8$

$m = \pm\sqrt[4]{8}$

median $= 1.68$

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Exercise D, Question 7

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{8}(x^2+1), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

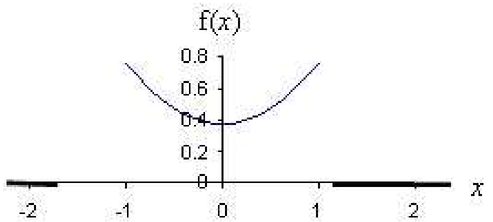
b What can you say about the mode of X?

c Write down the median value of X.

d Find the c.d.f. of X.

Solution:

a



b bimodal -1 and 1

c median $= 0$

d

Method 1	Method 2
$\int_{-1}^{x} \frac{3}{8}x^2 + \frac{3}{8}dx = \left[\frac{1}{8}x^3 + \frac{3}{8}x\right]_{-1}^{x}$ $= \left[\frac{1}{8}x^3 + \frac{3}{8}x\right] - \left[-\frac{1}{8} - \frac{3}{8}\right]$ $= \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2}$	$F(x) = \int \frac{3}{8}x^2 + \frac{3}{8}\,dx$ $= \frac{1}{8}x^3 + \frac{3}{8}x + C$ $F(1) = 1 \quad 1 = \frac{1}{8} + \frac{3}{8} + C$ $C = \frac{1}{2}$
$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2} & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$	

Solutionbank S2
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Exercise D, Question 8

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

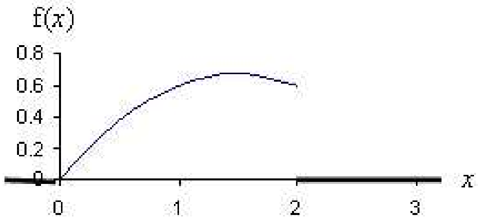
b Find the mode of X.

c Find the c.d.f. of X.

d Show that the median value of X lies between 1.23 and 1.24.

Solution:

a



b Find maximum by differentiating

$$\frac{d}{dx}\left(\frac{9}{10}x - \frac{3}{10}x^2\right) = \frac{9}{10} - \frac{6}{10}x$$

$$\frac{9}{10} - \frac{6}{10}x = 0$$

$$x = \frac{3}{2} \quad \text{mode} = 1.5$$

c

Method 1	Method 2
$\int_0^x \left(\frac{9}{10}t - \frac{3}{10}t^2\right) dt = \left[\frac{9}{20}t^2 - \frac{1}{10}t^3\right]_0^x$ $= \frac{9}{20}x^2 - \frac{1}{10}x^3$	$F(x) = \int \frac{9}{10}x - \frac{3}{10}x^2 dx$ $= \frac{9}{20}x^2 - \frac{1}{10}x^3 + C$ $F(2) = 1 \quad 1 = \frac{36}{20} - \frac{8}{10} + C$ $C = 0$
$F(x) \begin{cases} 0 & x < 0 \\ \frac{9}{20}x^2 - \frac{1}{10}x^3 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

d $F(1.23) = \frac{9}{20} \times 1.23^2 - \frac{1}{10} \times 1.23^3 = 0.495$

$F(1.24) = \frac{9}{20} \times 1.24^2 - \frac{1}{10} \times 1.24^3 = 0.501$

Since 0.5 is in between the median lies between 1.23 and 1.24.

Solutionbank S2

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Exercise D, Question 9

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{8}(x^2 - 1), & 1 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

a Find the p.d.f. of the random variable X.

b Find the mode of X.

c Find the median of X.

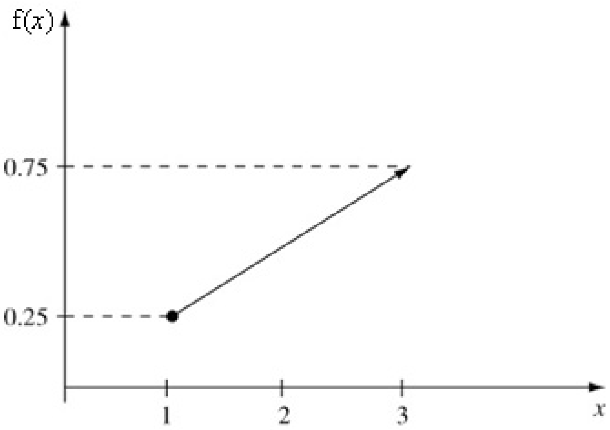
d Find the quartiles of X.

Solution:

a Differentiating $\frac{d}{dx}\left(\frac{1}{8}x^2 - \frac{1}{8}\right) = \frac{1}{4}x$

$$f(x) = \begin{cases} \frac{1}{4}x, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

b mode = 3



c $\frac{1}{8}m^2 - \frac{1}{8} = 0.5$

$\frac{1}{8}m^2 = \frac{5}{8}$

$m = \sqrt{5}$

median $= \sqrt{5}$

d $\frac{1}{8}Q_1^{\ 2} - \frac{1}{8} = 0.25$

$\frac{1}{8}Q_1^{\ 2} = \frac{3}{8}$

$Q_1 = \sqrt{3}$

lower quartile $= \sqrt{3}$

$\frac{1}{8}Q_3^{\ 2} - \frac{1}{8} = 0.75$

$\frac{1}{8}Q_3^{\ 2} = \frac{7}{8}$

$Q_3 = \sqrt{7}$

upper quartile $= \sqrt{7}$

Solutionbank S2
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Exercise D, Question 10

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 0, \\ 4x^3 - 3x^4, & 0 \le x \le 1, \\ 1, & x > 1. \end{cases}$$

a Find the p.d.f. of the random variable X.

b Find the mode of X.

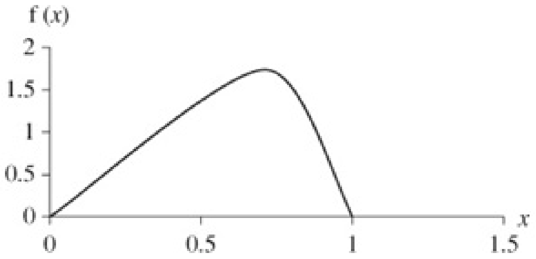
c Find $P(0.2 < X < 0.5)$.

Solution:

a $\frac{d}{dx}\left(4x^3 - 3x^4\right) = 12x^2 - 12x^3$

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

b



maximum $\frac{d}{dx}\left(12x^2 - 12x^3\right) = 24x - 36x^2$

$$24x - 36x^2 = 0$$
$$12x(2 - 3x) = 0$$
$$x = 0 \text{ or } \frac{2}{3} \text{ mode} = \frac{2}{3}$$

c $P(0.2 < X < 0.5) = F(0.5) - F(0.2)$

$$= \left(4 \times 0.5^3 - 3 \times 0.5^4\right) - \left(4 \times 0.2^3 - 3 \times 0.2^4\right)$$
$$= 0.2853$$

Solutionbank S2

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Exercise D, Question 11

Question:

The amount of vegetables eaten by a family in a week is a continuous random variable W kg. The continuous random variable W has p.d.f. given by

$$f(w) = \begin{cases} \frac{20}{5^5} w^3(5-w), & 0 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the c.d.f. of the random variable W.

b Find, to 3 decimal places, the probability that the family eat between 2 kg and 4 kg of vegetables in one week. ***E***

Solution:

a

Method 1	Method 2
$\int_0^w \frac{20}{5^5} t^3(5-t)\,dt = \left[\frac{100}{4\times5^5}t^4 - \frac{20}{5\times5^5}t^5\right]_0^w$ $= \frac{25}{5^5}w^4 - \frac{4}{5^5}w^5$ $= \frac{w^4}{5^5}(25-4w)$	$F(x) = \int \frac{20}{5^5} w^3(5-w)\,dx$ $= \frac{25}{5^5}w^4 - \frac{4}{5^5}w^5 + C$ $F(5) = 1 \quad 1 = 5 - 4 + C$ $C = 0$
$F(x)\begin{cases} 0 & x < 0 \\ \frac{w^4}{5^5}(25-4w) & 0 \le x \le 5 \\ 1 & x > 5 \end{cases}$	

b $P(2 < w < 4) = F(4) - F(2)$

$$= \left[\frac{4^4}{5^5}(25-16)\right] - \left[\frac{2^4}{5^5}(25-8)\right]$$

$$= 0.650$$

Solutionbank S2

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Exercise D, Question 12

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}, & 0 \le x < 1, \\ \frac{x^3}{5}, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the cumulative distribution function.

b Find, to 3 decimal places, the median and the inter-quartile range of the distribution. ***E***

Solution:

a Method 1

$$F(x) = \int_0^x \frac{1}{4}\,dt$$
$$= \left[\frac{t}{4}\right]_0^x$$
$$= \frac{x}{4}$$

$$\int_0^1 \frac{1}{4}\,dx + \int_1^x \frac{t^3}{5}\,dt = \left[\frac{x}{4}\right]_0^1 + \left[\frac{t^4}{20}\right]_1^x$$
$$= \frac{1}{4} + \frac{x^4}{20} - \frac{1}{20}$$

$$= \frac{x^4}{20} + \frac{1}{5}$$

Method 2

$$F(x) = \int \frac{1}{4}\,dx$$
$$= \frac{x}{4} + C$$

$F(0) = 0$ therefore $C = 0$

$$\int \frac{x^3}{5}dx = \frac{x^4}{20} + C$$

$F(2) = 1$ therefore

$$1 = \frac{16}{20} + C$$

$$C = \frac{1}{5}$$

$$F(x)\begin{cases} 0, & x < 0, \\ \frac{x}{4}, & 0 \le x < 1, \\ \frac{x^4}{20} + \frac{1}{5}, & 1 \le x \le 2, \\ 1, & x > 2 \end{cases}$$

b $\frac{Q_1^4}{20} + \frac{1}{5} = 0.25$

$Q_1^4 + 4 - 5 = 0$

$Q_1 = 1$

$\frac{Q_3^4}{20} + \frac{1}{5} = 0.75$

$Q_3^4 + 4 = 15$

$Q_3^4 = 11$

$Q_3 = 1.821$ or -1.821

Therefore upper quartile $= 1.821$ as -1.821 is not in range

$IQR = 1.82 - 1 = 0.821$

$\frac{m^4}{20} + \frac{1}{5} = 0.5$

$m^4 + 4 - 10 = 0$

$m^4 = 6$

$m = 1.57$

Therefore median $= 1.57$

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Exercise E, Question 1

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{3}\left(1+\frac{x}{2}\right), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a $E(X)$ and $E(3X+2)$,

b $Var(X)$ and $Var(3X+2)$,

c $P(X < 1)$,

d $P(X > \mu)$,

e $P(0.5 < X < 1.5)$.

Solution:

a $\mathrm{E}(X) = \int_0^2 \frac{x}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$

$= \int_0^2 \frac{x}{3} + \frac{x^2}{6} \, \mathrm{d}x$

$= \left[\frac{x^2}{6} + \frac{x^3}{18}\right]_0^2$

$= \left[\frac{2^2}{6} + \frac{2^3}{18}\right]$

$= \frac{10}{9}$

$\mathrm{E}(3X+2) = 3\,\mathrm{E}(X) + 2$

$= 3 \times \frac{10}{9} + 2$

$= 5\frac{1}{3}$

b $\mathrm{Var}(X) = \int_0^2 \frac{x^2}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x - \left(\frac{10}{9}\right)^2$

$= \int_0^2 \frac{x^2}{3} + \frac{x^3}{6} \, \mathrm{d}x - \left(\frac{100}{81}\right)$

$= \left[\frac{x^3}{9} + \frac{x^4}{24}\right]_0^2 - \left(\frac{100}{81}\right)$

$= \left[\frac{2^3}{9} + \frac{2^4}{24}\right] - \left(\frac{100}{81}\right)$

$= 0.321$

$\mathrm{Var}(3X+2) = 9\mathrm{Var}(X)$

$= 2.89$

c $\mathrm{P}(X < 1) = \int_0^1 \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$

$= \int_0^1 \frac{1}{3} + \frac{x}{6} \, \mathrm{d}x$

$= \left[\frac{x}{3} + \frac{x^2}{12}\right]_0^1$

$= \left[\frac{1}{3} + \frac{1}{12}\right]$

$= \frac{5}{12}$

d $\mathrm{P}(X > \mu) = \mathrm{P}(X > \frac{10}{9})$

$= 1 - \int_0^{\frac{10}{9}} \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$ or $\int_{\frac{10}{9}}^2 \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x = \left[\frac{x}{3} + \frac{x^2}{12}\right]_{\frac{10}{9}}^2$

$= 1 - \int_0^{\frac{10}{9}} \frac{1}{3} + \frac{x}{6} \, \mathrm{d}x$ $\quad = \left[\frac{2}{3} + \frac{4}{12}\right] - \left[\frac{10}{27} + \frac{100}{972}\right]$

$= 1 - \left[\frac{x}{3} + \frac{x^2}{12}\right]_0^{\frac{10}{9}}$ $\quad = \frac{128}{243}$

$= 1 - \left[\frac{10}{27} + \frac{100}{972}\right]$

$= 1 - \frac{115}{243}$

$= \frac{128}{243}$

e $\mathrm{P}(0.5 < X < 1.5) = \mathrm{P}(X < 1.5) - \mathrm{P}(X < 0.5)$

$= \int_{0.5}^{1.5} \left(\frac{1}{3} + \frac{x}{6}\right) \mathrm{d}x$

$= \left[\frac{x}{3} + \frac{x^2}{12}\right]_0^{1.5}$

$= \left[\frac{1.5}{3} + \frac{1.5^2}{12}\right] - \left[\frac{0.5}{3} + \frac{0.5^2}{12}\right]$

$= 0.5$

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Exercise E, Question 2

Question:

The random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} 2 - 2x, & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Evaluate E(X).

b Evaluate Var(X).

c Write down the values of $E(2X+1)$ and $Var(2X+1)$.

d Specify fully the cumulative distribution function of X.

e Work out the median value of X.

Solution:

a $E(X) = \int_0^1 2x - 2x^2 dx$

$= \left[x^2 - \frac{2}{3}x^3\right]_0^1$

$= \frac{1}{3}$

b $Var(X) = \int_0^1 2x^2 - 2x^3 dx - \left(\frac{1}{3}\right)^2$

$= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1 - \left(\frac{1}{9}\right)$

$= \frac{1}{18}$

c $E(2X + 1) = 2E(X) + 1$

$= 2 \times \frac{1}{3} + 1$

$= \frac{5}{3}$

$Var(2X + 1) = 4Var(X)$

$= \frac{4}{18} = \frac{2}{9}$

d

Method 1	Method 2
$\int_0^x (2 - 2t)\, dt = \left[2x - x^2\right]_0^x$ $= 2x - x^2$	$\int 2 - 2x dx = 2x - x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ 2x - x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$	

e $2x - x^2 = 0.5$

$x^2 - 2x + 0.5 = 0$

$x = \frac{2 \pm \sqrt{4 - 2}}{2}$

$x = 1.71$ or 0.293

median $= 0.293$ as 1.71 is not in the range

Solutionbank S2

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Exercise E, Question 3

Question:

The continuous random variable Y has cumulative distribution function given by

$$F(y) = \begin{cases} 0, & y < 1, \\ k(y^2 - y), & 1 \le y \le 2, \\ 1, & y > 2, \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{1}{2}$.

b Find $P(Y < 1.5)$.

c Find the value of the median.

d Specify fully the probability density function f(y).

Solution:

a $F(2) = 1$

$F(y) = k(y^2 - y)$

$k\ (4 - 2) = 1$

$k = \frac{1}{2}$

b $P(Y < 1.5) = F(1.5)$

$= \frac{1}{2} \times (1.5^2 - 1.5)$

$= 0.375$

c $\frac{1}{2}(y^2 - y) = 0.5$

$y^2 - y - 1 = 0$

$y = \frac{1 \pm \sqrt{1+4}}{2}$

$y = 1.62$ or -0.618

median $= 1.62$ as -0.618 is not in the range

d $\frac{d}{dx}\left[\frac{1}{2}(y^2 - y)\right] = y - \frac{1}{2}$

$$f(x) = \begin{cases} y - \frac{1}{2} & 1 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

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Exercise E, Question 4

Question:

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

a Find $P(X > 2.4)$.

b Find the median.

c Find the probability density function, $f(x)$.

d Evaluate $E(X)$.

e Find the mode of X.

Solution:

a $P(X > 2.4) = F(3) - F(2.4)$

$= \frac{1}{5}(3^2 - 4) - \frac{1}{5}(2.4^2 - 4)$

$= 0.648$

or

$P(X > 2.4) = 1 - F(2.4)$

$= 1 - \frac{1}{5}(2.4^2 - 4)$

$= 0.648$

b $\frac{1}{5}(x^2 - 4) = 0.5$

$2x^2 - 8 = 5$

$2x^2 = 13$

$x^2 = 6.5$

$x = 2.55$ or -2.55

median $= 2.55$ as -2.55 is not in the range

c $\frac{d}{dx}\left[\frac{1}{5}(x^2 - 4)\right] = \frac{2x}{5}$

$$f(x) = \begin{cases} \frac{2x}{5} & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

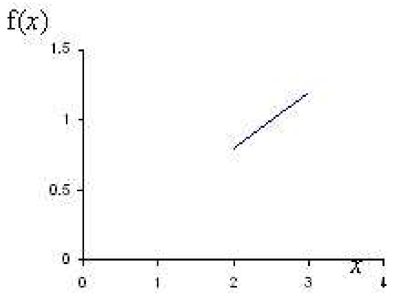
d $E(X) = \int_2^3 \frac{2x^2}{5} dx$

$= \left[\frac{2x^3}{15}\right]_2^3$

$= \frac{54}{15} - \frac{16}{15}$

$= \frac{38}{15}$

e



mode $= 3$

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Exercise E, Question 5

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx^2, & 0 \le x \le 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{8}$.

b Calculate $E(X)$.

c Specify fully the cumulative distribution function of X.

d Find the value of the median.

e Find the value of the mode.

Solution:

a $\int_0^2 kx^2\,dx = 1$

$$\left[\frac{kx^3}{3}\right]_0^2 = 1$$

$$\frac{8k}{3} = 1$$

$$k = \frac{3}{8}$$

b $E(X) = \int_0^2 \frac{3x^3}{8}\,dx$

$$= \left[\frac{3x^4}{32}\right]_0^2$$

$$= 1.5$$

c Method 1

$$F(x) = \int_0^x \frac{3t^2}{8}\,dt$$

$$= \left[\frac{t^3}{8}\right]_0^x$$

$$= \frac{x^3}{8}$$

Method 2

$$F(x) = \int \frac{3x^2}{8}\,dx$$

$$= \frac{x^3}{8} + C$$

$F(2) = 1$ therefore $\frac{8}{8} + C = 1$

$$C = 0$$

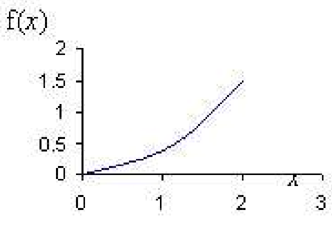
$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

d $\frac{m^3}{8} = 0.5$

$$m^3 = 4$$

$$m = 1.59$$

e



mode = 2

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Exercise E, Question 6

Question:

The random variable Y has probability density function $f(y)$ given by

$$f(x) = \begin{cases} k(y^2 + 2y + 2), & 1 \le y \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{62}$.

b Specify fully the cumulative distribution function of Y.

c Evaluate $P(Y \le 2)$.

Solution:

a
$$\int_1^3 k(y^2+2y+2)\,dy = 1$$
$$\left[k\left(\frac{y^3}{3}+y^2+2y\right)\right]_1^3 = 1$$
$$k\left(\frac{3^3}{3}+3^2+6\right)-k\left(\frac{1}{3}+1+2\right) = 1$$
$$\frac{62}{3}k = 1$$
$$k = \frac{3}{62}$$

b **Method 1**

$$F(y) = \int_1^y \frac{3}{62}(t^2+2t+2)\,dt$$
$$= \left[\frac{3}{62}\left(\frac{t^3}{3}+t^2+2t\right)\right]_1^y$$
$$= \frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)-\frac{3}{62}\left(\frac{1}{3}+1+2\right)$$
$$= \frac{y^3}{62}+\frac{3y^2}{62}+\frac{3y}{31}-\frac{5}{31}$$

Method 2

$$F(x) = \int \frac{3}{62}(y^2+2y+2)\,dx$$
$$= \frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)+C$$
$F(3) = 1$ therefore
$$\frac{3}{62}\left(\frac{3^3}{3}+3^2+6\right)+C = 1$$
$$C = -\frac{5}{31}$$

$$F(x) = \begin{cases} 0 & x<1 \\ \frac{y^3}{62}+\frac{3y^2}{62}+\frac{3y}{31}-\frac{5}{31} & 1\le x\le 3 \\ 1 & x>3 \end{cases}$$

c $P(Y<2) = \int_1^2 \frac{3}{62}(y^2+2y+2)\,dy$

$$= \left[\frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)\right]_1^2$$
$$= \frac{3}{62}\left(\frac{2^3}{3}+2^2+4\right)-\frac{3}{62}\left(\frac{1}{3}+1+2\right)$$
$$= \frac{11}{31}$$

or $F(2) = \frac{2^3}{62}+\frac{3\times 2^2}{62}+\frac{6}{31}-\frac{5}{31}$
$$= \frac{11}{31}$$

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Exercise E, Question 7

Question:

A random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} \frac{3}{32}(4 - x^2), & -2 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the probability density function of X.

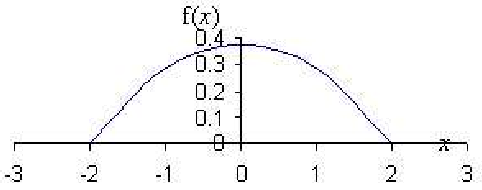
b Write down the mode of X.

c Specify fully the cumulative distribution function of X.

d Find $P(0.5 < X < 1.5)$.

Solution:

a



b mode = 0

c Method 1

$$F(x) = \int_{-2}^{x} \frac{3}{32}(4 - t^2)\,dt$$

$$= \left[\frac{12t}{32} - \frac{t^3}{32}\right]_{-2}^{x}$$

$$= \left(\frac{12x}{32} - \frac{x^3}{32}\right) - \left(-\frac{24}{32} + \frac{8}{32}\right)$$

$$= \frac{12x}{32} - \frac{x^3}{32} + \frac{1}{2}$$

Method 2

$$F(x) = \int \frac{3}{32}(4 - x^2)\,dx$$

$$= \frac{12x}{32} - \frac{x^3}{32} + C$$

$$F(2) = 1 \text{ therefore } \frac{24}{32} - \frac{8}{32} + C$$

=1

$$C = 0.5$$

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{12x}{32} - \frac{x^3}{32} + \frac{1}{2} & -2 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

d $P(0.5 < X < 1.5) = F(1.5) - F(0.5)$

$$= \left(\frac{18}{32} - \frac{1.5^3}{32} + \frac{1}{2}\right) - \left[\frac{6}{32} - \frac{0.5^3}{32} + \frac{1}{2}\right]$$

$$= \frac{35}{128} \text{ or } 0.273$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 8

Question:

A random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} \frac{1}{3}, & 0 \le x < 1, \\ \frac{2}{7}x^2, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find E(X).

b Specify fully the cumulative distribution function of X.

c Find the median of X.

Solution:

a $\mathrm{E}(X) = \int_0^1 \frac{x}{3}\mathrm{d}x + \int_1^2 \frac{2x^3}{7}\mathrm{d}x$

$= \left[\frac{x^2}{6}\right]_0^1 + \left[\frac{2x^4}{28}\right]_1^2$

$= \frac{1}{6} + \left(\frac{32}{28} - \frac{2}{28}\right)$

$= \frac{26}{21}$

b **Method 1**

$\mathrm{F}(x) = \int_0^x \frac{1}{3}\,\mathrm{d}t$

$= \left[\frac{t}{3}\right]_0^x$

$= \frac{x}{3}$

$\int_0^1 \frac{1}{3}\mathrm{d}x + \int_1^x \frac{2t^2}{7}\mathrm{d}t = \left[\frac{x}{3}\right]_0^1 + \left[\frac{2t^3}{21}\right]_1^x$

$= \frac{1}{3} + \frac{2x^3}{21} - \frac{2}{21}$

$= \frac{2x^3}{21} + \frac{5}{21}$

Method 2

$\mathrm{F}(x) = \int \frac{1}{3}\mathrm{d}x$

$= \frac{x}{3} + C$

$\mathrm{F}(0) = 0$ therefore $C = 0$

$\int \frac{2x^2}{7}\mathrm{d}x = \frac{2x^3}{21} + C$

$\mathrm{F}(2) = 1$ therefore

$1 = \frac{16}{21} + C$

$C = \frac{5}{21}$

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{3} & 0 \le x < 1 \\ \frac{2x^3}{21} + \frac{5}{21} & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

c $\mathrm{F}(1) = \frac{1}{3}$ therefore median lies in interval $1 \le x \le 2$

$\frac{2x^3}{21} + \frac{5}{21} = 0.5$

$2x^3 + 5 = 10.5$

$2x^3 = 5.5$

$x^3 = 2.75$

$x = 1.40$

median $= 1.40$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 9

Question:

A continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x)=\begin{cases} kx-k, & 1 \le x \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k=\frac{1}{2}$.

b Find $\mathrm{E}(X)$.

c Work out the cumulative distribution function, $\mathrm{F}(x)$.

d Show that the median value lies between 2.4 and 2.5.

Solution:

a $\int_1^3 kx - k\,dx = 1$

$$\left[\frac{kx^2}{2} - kx\right]_1^3 = 1$$

$$\left(\frac{9k}{2} - 3k\right) - \left(\frac{k}{2} - k\right) = 1$$

$$2k = 1$$

$$k = \frac{1}{2}$$

b $E(X) = \int_1^3 \frac{x^2}{2} - \frac{x}{2}dx$

$$= \left[\frac{x^3}{6} - \frac{x^2}{4}\right]_1^3$$

$$= \left(\frac{9}{4}\right) - \left(-\frac{1}{12}\right)$$

$$= \frac{7}{3}$$

c Method 1

$$F(x) = \int_1^x \frac{t}{2} - \frac{1}{2}dt$$

$$= \left[\frac{t^2}{4} - \frac{t}{2}\right]_1^x$$

$$= \left(\frac{x^2}{4} - \frac{x}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4}$$

Method 2

$$F(x) = \int \frac{x}{2} - \frac{1}{2}dx$$

$$= \frac{x^2}{4} - \frac{x}{2} + C$$

$F(3) = 1$ therefore $\frac{9}{4} - \frac{3}{2} + C = 1$

$$C = 0.25$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

d $F(2.4) = \frac{2.4^2}{4} - \frac{2.4}{2} + \frac{1}{4} = 0.5$

$$F(2.5) = \frac{2.5^2}{4} - \frac{2.5}{2} + \frac{1}{4} = 0.5$$

Since 0.5 lies in between, the median is between 2.4 and 2.5.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 10

Question:

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} x, & 0 \le x < 1, \\ \frac{3x^2}{14}, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the probability density function of X.

b Find the mode of X.

c Find $E(2X)$.

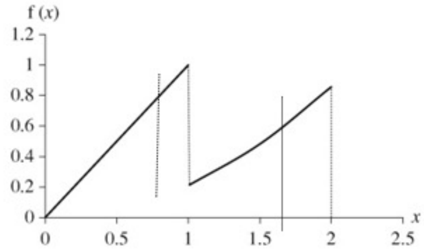
d Find $\text{Var}(2X + 1)$

e Specify fully the cumulative distribution function of X.

f Using your answer to part **e** find the median of X.

Solution:

a



b mode = 1

c $\text{E}(X) = \int_0^1 x^2 \text{d}x + \int_1^2 \frac{3x^3}{14}\text{d}x$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{3x^4}{56}\right]_1^2$

$= \frac{1}{3} + \left(\frac{48}{56} - \frac{3}{56}\right)$

$= \frac{191}{168}$

$\text{E}(2X) = 2 \times \frac{191}{168}$

$= \frac{191}{84}$

d $\text{Var}(X) = \int_0^1 x^3 \text{d}x + \int_1^2 \frac{3x^4}{14}\text{d}x - \left(\frac{191}{168}\right)^2$

$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{3x^5}{70}\right]_1^2 - \left(\frac{191}{168}\right)^2$

$= \frac{1}{4} + \left(\frac{48}{35} - \frac{3}{70}\right) - \left(\frac{191}{168}\right)^2$

$= 0.28601$

$\text{Var}(2X+1) = 4 \times 0.286$
$= 1.14$

e Method 1

$\text{F}(x) = \int_0^x t \, \text{d}t$

$= \left[\frac{t^2}{2}\right]_0^x$

$= \frac{x^2}{2}$

$\int_0^1 t \, \text{d}t + \int_1^x \frac{3t^2}{14}\text{d}t = \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{t^3}{14}\right]_1^x$

$= \frac{1}{2} + \frac{x^3}{14} - \frac{1}{14}$

$= \frac{x^3}{14} + \frac{3}{7}$

Method 2

$\text{F}(x) = \int x \text{d}x$

$= \frac{x^2}{2} + C$

$\text{F}(0) = 0$ therefore $C = 0$

$\int \frac{3x^2}{14}\text{d}x = \frac{x^3}{14} + C$

$\text{F}(2) = 1$ therefore

$1 = \frac{8}{14} + C$

$C = \frac{3}{7}$

$$\text{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x < 1 \\ \frac{x^3}{14} + \frac{3}{7} & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

f $\frac{x^2}{2} = 0.5$ median = 1

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

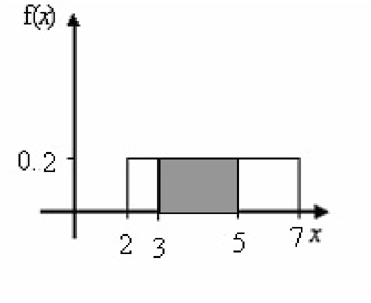
The continuous random variable $X \sim \mathrm{U}[2, 7]$.
Find

a $\mathrm{P}(3 < X < 5)$,

b $\mathrm{P}(X > 4)$.

Solution:

a $\frac{1}{b-a} = \frac{1}{7-2} = 0.2$



$\mathrm{P}(3 < X < 5) = (5-3) \times 0.2$
$= 0.4$

b $\mathrm{P}(X > 4) = (7-4) \times 0.2$
$= 0.6$

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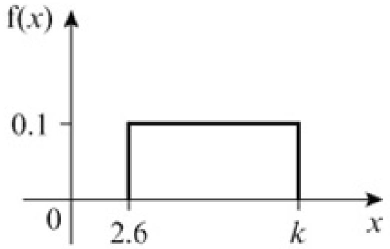
Exercise A, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

a the value of k,

b $P(4 < X < 7.9)$.

Solution:

a
$$\text{Area} = 1$$
$$(k - 2.6) \times 0.1 = 1$$
$$(k - 2.6) = 10$$
$$k = 12.6$$

b
$$P(4 < X < 7.9) = (7.9 - 4) \times 0.1$$
$$= 0.39$$

Solutionbank S2

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Exercise A, Question 3

Question:

The continuous random variable X has p.d.f.

$$f(x) = \begin{cases} k, & -2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a the value of k,

b $P(-1.3 < X < 4.2)$.

Solution:

a
$$\text{Area} = 1$$
$$k \times (6 - (-2)) = 1$$
$$8k = 1$$
$$k = \frac{1}{8}$$

b
$$P(-1.3 < X < 4.2) = \frac{1}{8} \times (4.2 - (-1.3))$$
$$= 0.6875$$

Solutionbank S2

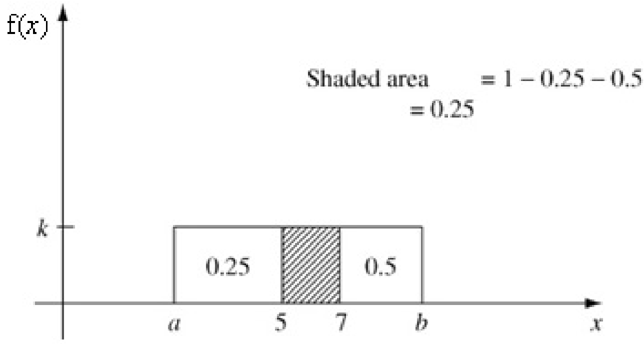
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

The continuous random variable $Y \sim \text{U}[a, b]$. Given that $\text{P}(Y < 5) = \frac{1}{4}$ and $\text{P}(Y > 7) = \frac{1}{2}$, find the value of a and the value of b.

Solution:



$$\text{shaded area} = \frac{1}{4}$$
$$2 \times k = \frac{1}{4}$$
$$k = \frac{1}{8}$$
$$(b - 7) \times \frac{1}{8} = 0.5$$
$$(b - 7) = 4$$
$$b = 11$$
$$(5 - a) \times \frac{1}{8} = 0.25$$
$$(5 - a) = 2$$
$$a = 3$$

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Exercise A, Question 5

Question:

The continuous random variable $X \sim \mathrm{U}[2, 8]$.

a Write down the distribution of $Y = 2X + 5$.

b Find $\mathrm{P}(12 < Y < 20)$.

Solution:

a

$2 \times 2 + 5 = 9$

$2 \times 8 + 5 = 21$

$Y \sim \mathrm{U}[9, 21]$

b For $Y, \dfrac{1}{b-a} = \dfrac{1}{21-9} = \dfrac{1}{12}$

$P(12 < Y < 20) = (20 - 12) \times \frac{1}{12}$

$= \frac{2}{3}$

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Exercise B, Question 1

Question:

The continuous variable Y is uniformly distributed over the interval $[-3, 5]$.

Find:

a $\mathrm{E}(X)$,

b $\mathrm{Var}(X)$,

c $\mathrm{E}(X^2)$,

d the cumulative distribution function of X, for all x.

Solution:

a $$\mathrm{E}(X) = \frac{5+(-3)}{2} = 1$$

b $$\mathrm{Var}(X) = \frac{\left(5-(-3)\right)^2}{12} = 5\frac{1}{3}$$

c $$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$$

$$5\frac{1}{3} = \mathrm{E}(X^2) - 1$$

$$\mathrm{E}(X^2) = 6\frac{1}{3}$$

d $$\mathrm{F}(x) = \int_{-3}^{x} \frac{1}{5-(-3)}\,\mathrm{d}t = \left[\frac{t}{8}\right]_{-3}^{x} = \frac{x+3}{8}$$

$$\mathrm{F}(x) = \begin{cases} 0 & x < -3 \\ \dfrac{x+3}{8} & -3 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

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Exercise B, Question 2

Question:

Find E(X) and Var(X) for the following probability density functions.

a $f(x) = \begin{cases} \frac{1}{4}, & 1 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$

b $f(x) = \begin{cases} \frac{1}{8}, & -2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a $E(X) = \frac{5+1}{2}$

$= 3$

$\text{Var}(X) = \frac{(5-1)^2}{12}$

$= 1\frac{1}{3}$

b $E(X) = \frac{6+(-2)}{2}$

$= 2$

$\text{Var}(X) = \frac{(6-(-2))^2}{12}$

$= 5\frac{1}{3}$

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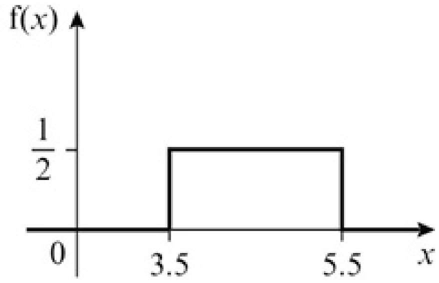
Exercise B, Question 3

Question:

The continuous random variable X has p.d.f as shown in the diagram.



Find:

a $E(X)$,

b $Var(X)$,

c $E(X^2)$,

d the cumulative distribution function of X, for all x.

Solution:

a $\mathrm{E}(X) = \dfrac{5.5+3.5}{2}$

$= 4.5$

b $\mathrm{Var}(X) = \dfrac{(5.5-3.5)^2}{12}$

$= \dfrac{1}{3}$

c $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$\dfrac{1}{3} = \mathrm{E}(X^2) - 20.25$

$\mathrm{E}(X^2) = 20\dfrac{7}{12} = 20.6 \text{ (3 s.f.)}$

d $\mathrm{F}(x) = \displaystyle\int_{3.5}^{x} \frac{1}{5.5-3.5}\,\mathrm{d}t$

$= \left[\dfrac{t}{2}\right]_{3.5}^{x}$

$= \dfrac{x}{2} - \dfrac{3.5}{2}$

$= \dfrac{x}{2} - 1.75$

$$\mathrm{F}(x) = \begin{cases} 0 & x < 3.5 \\ \dfrac{x}{2} - 1.75 & 3.5 \le x \le 5.5 \\ 1 & x > 5.5 \end{cases}$$

Solutionbank S2
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Exercise B, Question 4

Question:

The continuous random variable $Y \sim \text{U}[a, b]$. Given $\text{E}(Y) = 1$ and $\text{Var}(Y) = \frac{4}{3}$, find the value of a and the value of b.

Solution:

$$\text{E}(Y)$$
$$\frac{a+b}{2} = 1$$
$$a + b = 2 \qquad (1)$$

$$\text{Var}(Y)$$
$$\frac{(b-a)^2}{12} = \frac{4}{3}$$
$$(b-a)^2 = 16 \qquad (2)$$

Solving equations (**1**) and (**2**) simultaneously

$$b = 2 - a$$
$$(2 - a - a)^2 = 16$$
$$(2 - 2a) = \pm 4$$

$$2 - 2a = 4 \qquad\qquad 2 - 2a = -4$$
$$a = -1 \qquad\qquad a = 3$$
$$b = 2 - (-1) \qquad\qquad b = 2 - 3$$
$$= 3 \qquad\qquad = -1$$

Since $a < b$ $a = -1$ and $b = 3$

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Exercise B, Question 5

Question:

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{6}, & -1 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

Given that $Y = 4X - 6$, find $\text{E}(Y)$ and $\text{Var}(Y)$.

Solution:

$$\text{E}(X) = \frac{5+(-1)}{2}$$
$$= 2$$
$$\text{Var}(X) = \frac{(5-(-1))^2}{12}$$
$$= 3$$

$$\text{E}(Y) = 4\text{E}(X) - 6$$
$$= 8 - 6$$
$$= 2$$

$$\text{Var}(Y) = 16\ \text{Var}(X)$$
$$= 48$$

Solutionbank S2

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Exercise B, Question 6

Question:

The random variable X is the length of a side of a square. $X \sim U[4.5, 5.5]$.
The random variable Y is the area of the square.
Find $E(Y)$.

Solution:

$E(Y) = E(X^2)$ or $\int_{4.5}^{5.5} x^2 dx = \left[\frac{x^3}{3}\right]_{4.5}^{5.5}$

$= \left[\frac{5.5^3}{3}\right] - \left[\frac{4.5^3}{3}\right]$

$= 25\frac{1}{12}$

$E(X) = \frac{4.5+5.5}{2}$

$= 5$

$Var(X) = \frac{(5.5-4.5)^2}{12}$

$= \frac{1}{12}$

$Var(X) = E(X^2) - (E(X))^2$

$\frac{1}{12} = E(X^2) - 25$

$E(X^2) = 25\frac{1}{12}$

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Exercise B, Question 7

Question:

In a computer game an alien appears every 2 seconds. The player stops the alien by pressing a key. The object of the game is to stop the alien as soon as it appears. Given that the player actually presses the key T s after the alien first appears, a simple model of the game assumes that T is a continuous uniform random variable defined over the interval [0, 1].

a Write down $P(T < 0.2)$.

b Write down $E(T)$.

c Use integration to find Var (T).

Solution:

$$\frac{1}{b-a} = \frac{1}{1-0} = 1$$

a $P(T < 0.2) = (0.2 - 0) \times 1$
$= 0.2$

b $E(T) = 0.5$

c $\mathrm{Var}(T) = \int_0^1 t^2 \, dt - 0.5^2$
$= \left[\frac{t^3}{3}\right]_0^1 - 0.25$
$= \frac{1}{12}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

The continuous random variable X is uniformly distributed over the interval $[-2, 5]$.

a Sketch the probability density function $f(x)$ of X.

Find

b $E(X)$,

c $Var(X)$,

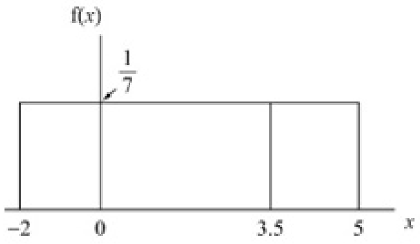
d the cumulative distribution function of X, for all x,

e $P(3.5 < X < 5.5)$,

f $P(X = 4)$.

Solution:

a



b $E(X) = \frac{5+(-2)}{2} = 1.5$

c $\text{Var}(X) = \frac{(5-(-2))^2}{12} = 4\frac{1}{12}$

d $F(x) = \int_{-2}^{x} \frac{1}{5-(-2)}\, dt$

$= \left[\frac{t}{7}\right]_{-2}^{x}$

$= \frac{x+2}{7}$

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{x+2}{7} & -2 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

e $P(3.5 < X < 5.5) = P(3.5 < X < 5)$

$= ((5 - 3.5) \times \frac{1}{7})$

$= \frac{3}{14}$

f $P(X = 4) = 0$

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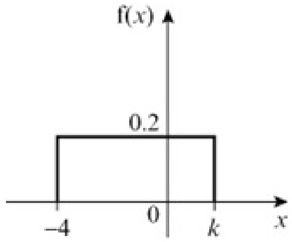
Exercise C, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

a the value of k,

b $P(-2 < X < -1)$,

c $E(X)$,

d $Var(X)$,

e the cumulative distribution function of X, for all x.

Solution:

a Area = 1 so

$$(k+4)\times 0.2 = 1$$
$$0.2k + 0.8 = 1$$
$$k = 1$$

b $P(-2 < X < -1) = 1 \times 0.2 = 0.2$

c $E(X) = \dfrac{-4+1}{2} = -1.5$

d $Var(X) = \dfrac{(b-a)^2}{12} = \dfrac{(1-(-4))^2}{12} = 2\frac{1}{12}$

e

$$F(x) = \int_{-4}^{x} \frac{1}{1-(-4)}\,dt$$
$$= \left[\frac{t}{5}\right]_{-4}^{x}$$
$$= \frac{x+4}{5}$$

$$F(x) = \begin{cases} 0 & x < -4 \\ \dfrac{x+4}{5} & -4 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

The continuous random variable Y is uniformly distributed on the interval $a \le Y \le b$.
Given $E(Y) = 2$ and $Var(Y) = 3$.
Find

a the value of a and the value of b,

b $P(X > 1.8)$.

Solution:

a $E(Y) = \frac{a+b}{2} = 2$ so $a + b = 4$ so $a = 4 - b$

$Var(Y) = \frac{(b-a)^2}{12} = 3$

Substituting for a gives $(2b-4)^2 = 36$

$(2b-4) = \pm 6$

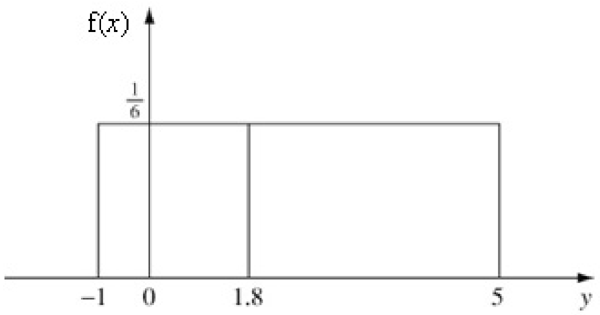
$b = 5$ or $b = -1$

$a = -1$ $a = 5$

but $b > a$

$b = 5$ $a = -1$

b



$P(X > 1.8) = (5 - 1.8) \times \frac{1}{6} = 0.533$ (3 s.f)

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Exercise C, Question 4

Question:

A child has a pair of scissors and a piece of string 20 cm long, which has a mark on one end. The child cuts the string, at a randomly chosen point, into two pieces. Let X represent the length of the piece of string with the mark on it.

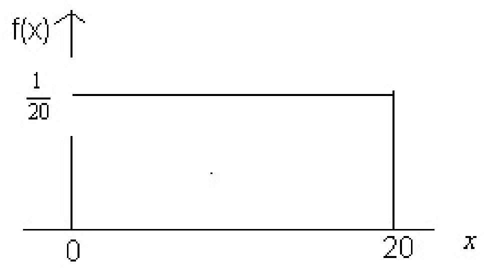
a Write down the name of the probability distribution of X and sketch the graph of its probability density function.

b Find the values of E(X) and Var(X).

c Using your model, calculate the probability that the shorter piece of string is at least 8 cm long.

Solution:

a $X \sim U(0, 20)$



b

$$E(X) = \frac{20+0}{2} = 10$$

$$\text{Var}(X) = \frac{(20-0)^2}{12} = \frac{400}{12} = 33\frac{1}{3}$$

c $P(8 \leq X \leq 12) = (12-8) \times \frac{1}{20} = 0.2$

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Exercise C, Question 5

Question:

Joan records the temperature every day. The highest temperature she recorded was 29 °C to the nearest degree. Let X represent the error in the measured temperature.

a Suggest a suitable model for the distribution of X.

b Using your model calculate the probability that the error will be less than 0.2 °C.

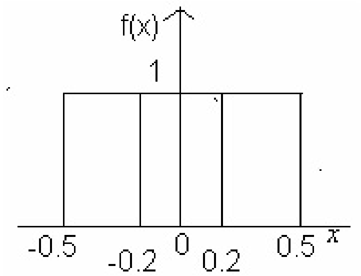
c Find the variance of the error in the measured temperature.

Solution:

a

$X \sim \text{U}(-0.5, 0.5)$

b



$\text{P}(-0.2 < X < 0.2) = 0.4 \times 1 = 0.4$

c $\text{Var}(X) = \frac{(b-a)^2}{12} = \frac{(0.5-(-0.5))^2}{12} = \frac{1}{12}$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 6

Question:

Jameil catches a bus to work every morning. According to the timetable the bus is due at 9 a.m., but Jameil knows that the bus can arrive at a random time between three minutes early and ten minutes late. The random variable X represents the time, in minutes, after 9 a.m. when the bus arrives.

a Suggest a suitable model for the distribution of X and specify it fully.

b Calculate the mean value of X.

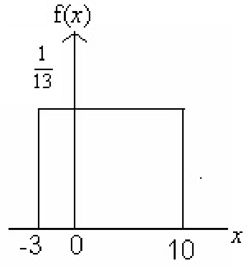
c Find the cumulative distribution function of X.

Jameil will be late for work if the bus arrives after 9.05 a.m.

d Find the probability that Jameil is late for work.

Solution:

a



$X \sim \mathrm{U}(-3, 10)$

$$f(x) = \begin{cases} \frac{1}{13} & -3 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

b $\text{Mean} = \mathrm{E}(X) = \frac{-3+10}{2} = 3.5$ minutes

c

$$F(x) = \int_{-3}^{x} \frac{1}{13}\, dt$$

$$= \left[\frac{t}{13}\right]_{-3}^{x}$$

$$= \frac{x+3}{13}$$

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{13} & -3 \le x \le 10 \\ 1 & x > 10 \end{cases}$$

d $P(5 < X < 10) = (10 - 5) \times \frac{1}{13} = \frac{5}{13}$

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Exercise C, Question 7

Question:

A plumber measures, to the nearest cm, the lengths of pipes.

a Suggest a suitable model to represent the difference between the true lengths and the measured lengths.

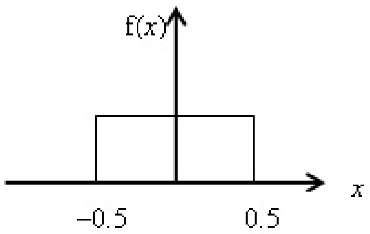
b Find the probability that for a randomly chosen rod the measured length will be within 0.2 cm of the true length.

c Three pipes are selected at random. Find the probability that all three pipes will be within 0.2 cm of the true length.

Solution:

a U(−0.5, 0.5)

b



$P(-0.2 \leq X \leq 0.2) = (0.2 - (-0.2)) \times 1 = 0.4$

c P(3 pipes between −0.2 and 0.2) $= 0.4^3 = 0.064$

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Exercise C, Question 8

Question:

A coffee machine dispenses coffee into cups. It is electronically controlled to cut off the flow of coffee randomly between 190 ml and 210 ml. The random variable X is the volume of coffee dispensed into a cup.

a Specify the probability density function of X and sketch its graph.

b Find the probability that the machine dispenses

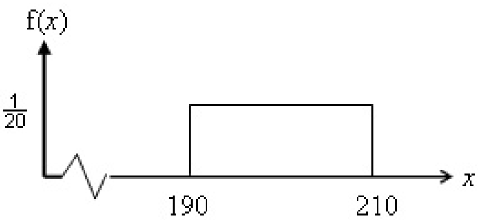
i less than 198 ml,

ii exactly 198 ml.

c Calculate the inter-quartile range of X.

Solution:

a



$$f(x) = \begin{cases} \frac{1}{20} & 190 \le x \le 210 \\ 0 & \text{otherwise} \end{cases}$$

b i $P(X < 198) = (198 - 190) \times \frac{1}{20} = \frac{2}{5}$

ii 0

c $\frac{210-190}{4} = 5$ is one quarter of the range

$Q_1 = 190 + 5 = 195$ $\quad Q_3 = 210 - 5 = 205$

$IQR = 205 - 195 = 10$

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Exercise C, Question 9

Question:

Write down the name of the distribution you would recommend as a suitable model for each of the following situations.

a the difference between the true height and the height measured, to the nearest cm, of randomly chosen people.

b the heights of randomly selected 18-year-old females.

Solution:

a Uniform

b Normal

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Exercise A, Question 1

Question:

The discrete random variable X takes integer values and is to be approximated by a normal distribution. Apply a continuity correction to the following probabilities.

a $P(X \leq 7)$

b $P(X < 10)$

c $P(X > 5)$

d $P(X \geq 3)$

e $P(17 \leq X \leq 20)$

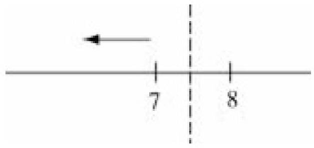
f $P(18 < X < 30)$

g $P(28 < X \leq 40)$

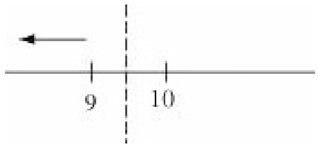
h $P(23 \leq X < 35)$

Solution:

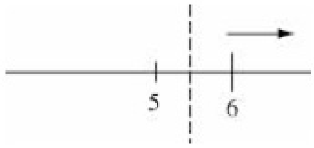
a $P(X \leq 7) = P(X \leq 7.5)$



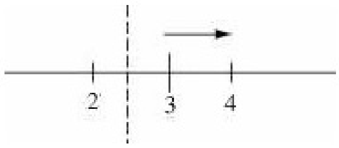
b $P(X < 10) = P(X \leq 9.5)$



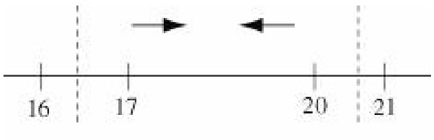
c $P(X > 5) = P(X \geq 5.5)$



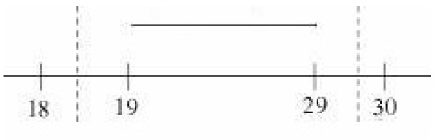
d $P(X \geq 3) = P(X \geq 2.5)$



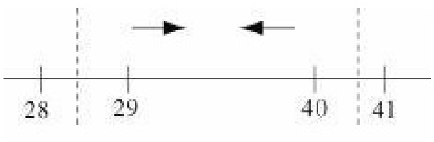
e $P(17 \leq X \leq 20) = P(16.5 \leq X < 20.5)$



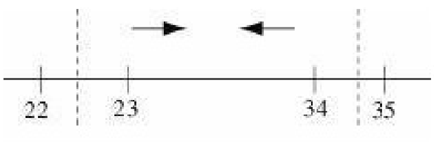
f $P(18 < X < 30) = P(18.5 \leq X < 29.5)$



g $P(28 < X \leq 40) = P(28.5 \leq X < 40.5)$



h $P(23 \leq X < 35) = P(22.5 \leq X < 34.5)$



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Exercise B, Question 1

Question:

The random variable $X \sim B(150, \frac{1}{3})$. Use a suitable approximation to estimate

a $P(X \leq 40)$,

b $P(X > 60)$,

c $P(45 \leq X \leq 60)$.

Solution:

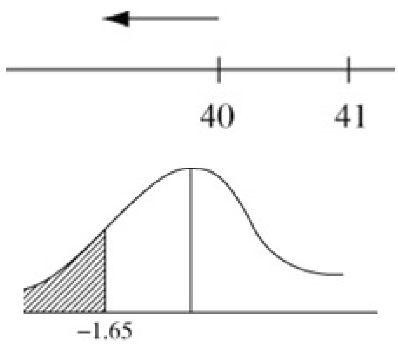
$X \sim B(150, \frac{1}{3})$

$Y \sim N(50, \sqrt{\frac{100}{3}}^2)$

$X \approx Y$

a

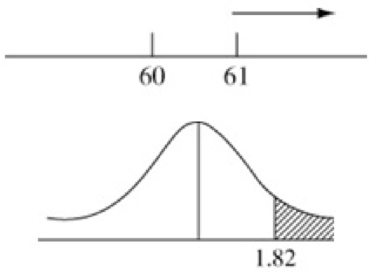
$$P(X \leq 40) \approx P(Y < 40.5)$$
$$= P\left(Z < \frac{40.5 - 50}{\sqrt{\frac{100}{3}}}\right)$$
$$= P(Z < -1.645\ldots)$$
$$= 1 - 0.9505$$
$$= 0.0495$$



$(\text{calc} = 0.0499\ldots)$

b

$$P(X > 60) \approx P(Y > 60.5)$$
$$= P(Z > 1.818\ldots)$$
$$= 1 - 0.9656$$
$$= 0.0344$$



$(\text{calc} = 0.03448\ldots)$

So accept awrt 0.0340 (3 s.f.)

c

$$P(45 \leq X \leq 60) \approx P(44.5 \leq Y < 60.5)$$
$$= P(-0.95 \leq Z < 1.82)$$
$$= 0.9656 - (1 - 0.8289)$$
$$= 0.7945$$

$(\text{calc} = 0.79512\ldots)$

accept awrt 0.795

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Exercise B, Question 2

Question:

The random variable $X \sim \mathrm{B}(200, 0.2)$. Use a suitable approximation to estimate

a $\mathrm{P}(X < 45)$,

b $\mathrm{P}(25 \leq X < 35)$,

c $\mathrm{P}(X = 42)$.

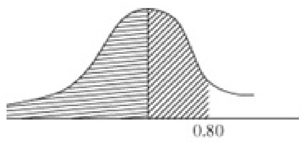
Solution:

$X \sim \mathrm{B}(200, 0.2)$

$Y \sim \mathrm{N}(40, \sqrt{32}^2)$ $\qquad X \approx Y$

a

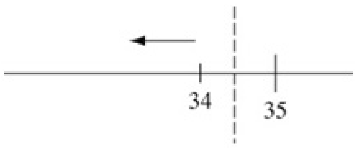
$$\begin{aligned}\mathrm{P}(X < 45) &\approx \mathrm{P}(Y \leq 44.5)\\ &= \mathrm{P}\left(Z < \frac{44.5-40}{\sqrt{32}}\right)\\ &= \mathrm{P}(Z < 0.7954\ldots)\\ &= 0.7881\end{aligned}$$

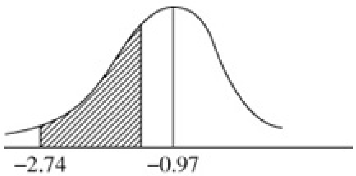


$(\text{calc} = 0.7868\ldots)$
So accept awrt $0.788 \sim 0.787$

b

$$\begin{aligned}\mathrm{P}(25 \leq X < 35) &= \mathrm{P}(24.5 \leq Y < 34.5)\\ &= \mathrm{P}(-2.74 \leq Z < -0.97\ldots)\\ &= [1-0.8340]-[1-0.9970]\\ &= 0.163\end{aligned}$$

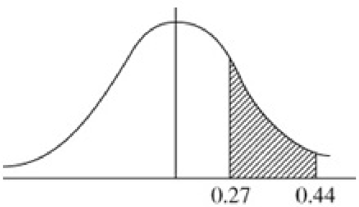




$(\text{calc} = 0.162386\ldots)$
so accept awrt $0.162 \sim 0.163$

c

$$\begin{aligned}\mathrm{P}(X = 42) &= \mathrm{P}(41.5 \leq Y < 42.5)\\ &= \mathrm{P}(0.265\ldots \leq Z < 0.4419\ldots)\\ &= 0.6700 - 0.6064\\ &= 0.0636\end{aligned}$$



$(\text{calc} = 0.066175\ldots)$
so accept awrt $0.0640 \sim 0.0660$

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Exercise B, Question 3

Question:

The random variable $X \sim B(100, 0.65)$. Use a suitable approximation to estimate

a $P(X > 58)$,

b $P(60 < X \leq 72)$,

c $P(X = 70)$.

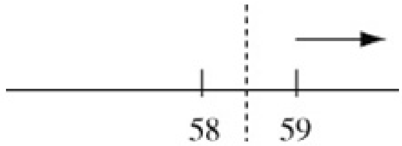
Solution:

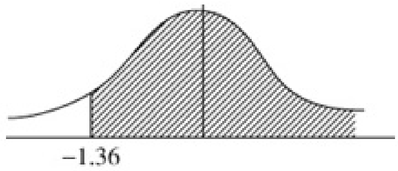
$X \sim \mathrm{B}(100, 0.65)$

$Y \sim \mathrm{N}(65, \sqrt{22.75}^2)$ $\qquad X \approx Y$

a

$$\begin{aligned} \mathrm{P}(X > 58) &\approx \mathrm{P}(Y > 58.5) \\ &= \mathrm{P}(Z > -1.36\ldots) \\ &= 0.9131 \end{aligned}$$

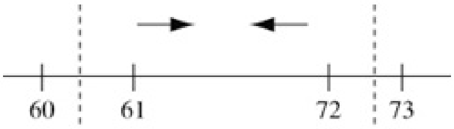




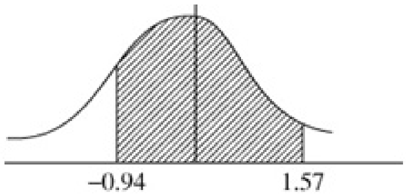
$(\text{calc} = 0.9135\ldots)$

So accept awrt $0.913 \sim 0.914$

b



$$\begin{aligned} \mathrm{P}(60 < X \le 72) &\approx \mathrm{P}(60.5 \le Y < 72.5) \\ &= \mathrm{P}(-0.94 \le Z < 1.57\ldots) \\ &= 0.9418 - (1 - 0.8264) \\ &= 0.7682 \end{aligned}$$

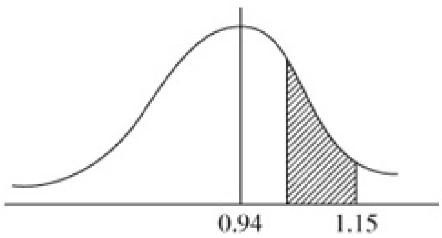


$(\text{calc} = 0.76935\ldots)$

So accept awrt $0.768 \sim 0.769$

c

$$\begin{aligned} \mathrm{P}(X = 70) &\approx \mathrm{P}(69.5 \le Y < 70.5) \\ &= \mathrm{P}(0.943 \le Z < 1.153\ldots) \\ &= 0.8749 - 0.8264 \\ &= 0.0485 \end{aligned}$$



$(\text{calc} = 0.04829\ldots)$

So accept awrt $0.0483 \sim 0.0485$

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Exercise B, Question 4

Question:

Sarah rolls a fair die 90 times. Use a suitable approximation to estimate the probability that the number of sixes she obtains is over 20.

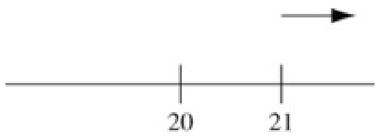
Solution:

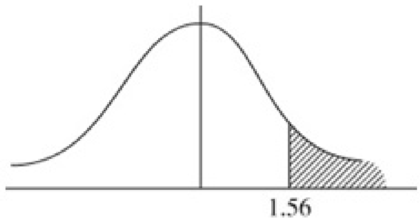
$X =$ number of sixes in 90 rolls

$X \sim \mathrm{B}(90, \frac{1}{6})$

$Y \sim \mathrm{N}(15, \sqrt{12.5}^2)$ $X \approx Y$

$$\begin{aligned} \mathrm{P}(X > 20) &\approx \mathrm{P}(Y \geq 20.5) \\ &= \mathrm{P}\left(Z \geq \frac{20.5-15}{\sqrt{12.5}}\right) \\ &= \mathrm{P}(Z \geq 1.555\ldots) \\ &= 1-0.9406 \\ &= 0.0594 \end{aligned}$$





$(\text{calc} = 0.059897\ldots)$
So accept awrt $0.059 \sim 0.060$

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Exercise B, Question 5

Question:

In a multiple choice test there are 4 possible answers to each question. Given that there are 60 questions on the paper, use a suitable approximation to estimate the probability of getting more than 20 questions correct if the answer to each question is chosen at random from the 4 available choices for each question.

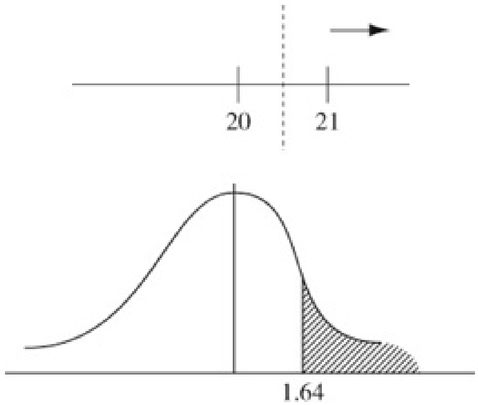
Solution:

$X =$ number of correct answers

$X \sim \mathrm{B}(60, \frac{1}{4})$

$Y \sim \mathrm{N}(15, \sqrt{11.25}^2)$ $\quad X \approx Y$

$$\begin{aligned} \mathrm{P}(X > 20) &\approx \mathrm{P}(Y \geq 20.5) \\ &= \mathrm{P}\left(Z \geq \frac{20.5-15}{\sqrt{11.25}}\right) \\ &= \mathrm{P}(Z \geq 1.639\ldots) \\ &= 1-0.9495 \\ &= 0.0505 \end{aligned}$$



(calc gives 0.050525…)
So accept awrt 0.0505

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Exercise B, Question 6

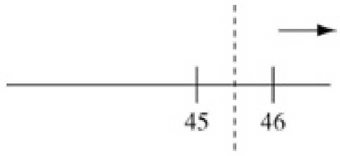
Question:

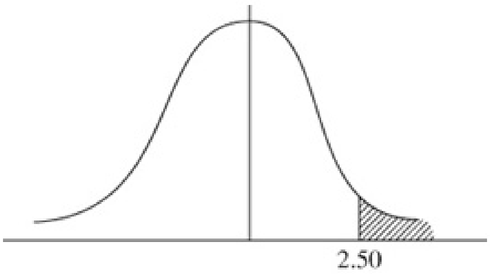
A fair coin is tossed 70 times. Use a suitable approximation to estimate the probability of obtaining more than 45 heads.

Solution:

$X =$ number of heads in 70 tosses of a fair coin
$X \sim \mathrm{B}(70, 0.5)$
$Y \sim \mathrm{N}(35, \sqrt{17.5}^2)$ $\quad X \approx Y$

$$\begin{aligned} \mathrm{P}(X > 45) &= \mathrm{P}(Y \geq 45.5) \\ &= \mathrm{P}\left(Z > \frac{45.5 - 35}{\sqrt{17.5}}\right) \\ &= \mathrm{P}(Z > 2.5099\ldots) \\ &= 1 - 0.9940 \\ &= 0.0060 \end{aligned}$$





(calc gives 0.006036...)
So accept awrt 0.006

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Exercise C, Question 1

Question:

The random variable $X \sim \text{Po}(30)$. Use a suitable approximation to estimate

a $\text{P}(X \leq 20)$,

b $\text{P}(X > 43)$,

c $\text{P}(25 \leq X \leq 35)$.

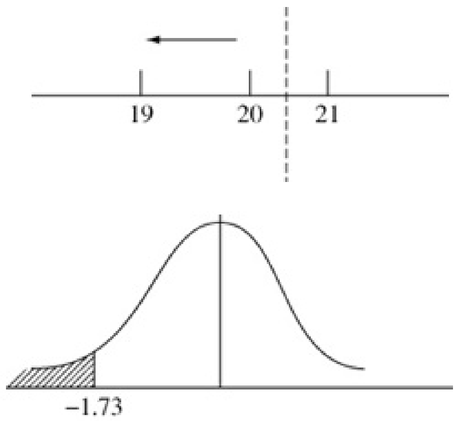
Solution:

$X \sim \text{Po}(30)$

$Y \sim \text{N}(30, \sqrt{30})$ $\quad X \approx Y$

a

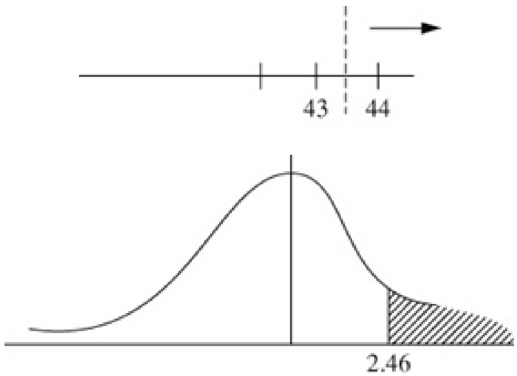
$$\begin{aligned} \text{P}(X \le 20) &\approx \text{P}(Y \le 20.5) \\ &= \text{P}\left(Z < \frac{20.5 - 30}{\sqrt{30}}\right) \\ &= \text{P}(Z < -1.7344\ldots) \\ &= 1 - 0.9582 \\ &= 0.0418 \end{aligned}$$



(calc gives 0.041418...)
So accept awrt 0.0410 ~ 0.0420

b

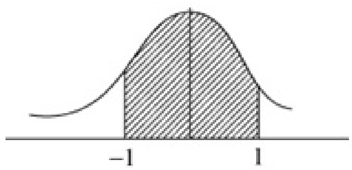
$$\begin{aligned} \text{P}(X > 43) &\approx \text{P}(Y > 43.5) \\ &= \text{P}\left(Z > \frac{43.5 - 30}{\sqrt{30}}\right) \\ &= \text{P}(Z > 2.46\ldots) \\ &= 1 - 0.9931 \\ &= 0.0069 \end{aligned}$$



(calc gives 0.006855...)
accept awrt 0.0069

c

$$\begin{aligned} \text{P}(25 \le X \le 35) &\approx \text{P}(24.5 \le Y < 35.5) \\ &= \text{P}(-1.00 < Z < 1.00\ldots) \\ &= 2 \times 0.3413 \\ &= 0.6826 \end{aligned}$$



(calc gives 0.68469...)
accept awrt 0.683 ~ 0.685

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Exercise C, Question 2

Question:

The random variable $X \sim \text{Po}(45)$. Use a suitable approximation to estimate

a $\text{P}(X < 40)$,

b $\text{P}(X \geq 50)$,

c $\text{P}(43 < X \leq 52)$.

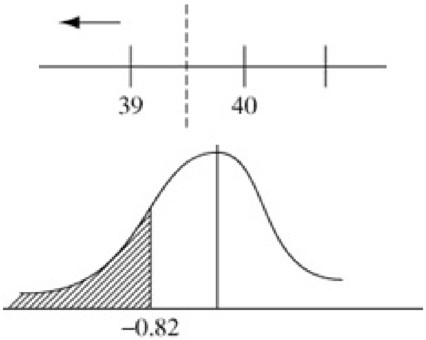
Solution:

$X \sim \text{Po}(45)$

$Y \sim \text{N}(45, \sqrt{45})$ $\qquad X \approx Y$

a

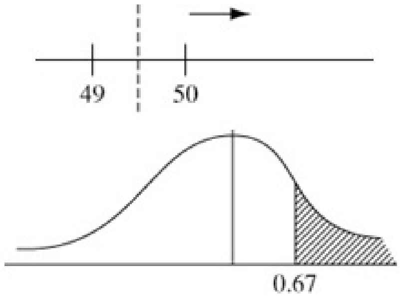
$$\begin{aligned} \text{P}(X < 40) &\approx \text{P}(Y \leq 39.5) \\ &= \text{P}\left(Z < \frac{39.5 - 45}{\sqrt{45}}\right) \\ &= \text{P}(Z < -0.819\ldots) \\ &= 1 - 0.7939 \\ &= 0.2061 \end{aligned}$$



(calc gives 0.20613…)
So accept awrt 0.206

b

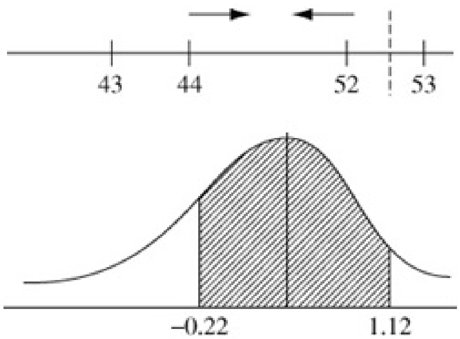
$$\begin{aligned} \text{P}(X \geq 50) &\approx \text{P}(Y \geq 49.5) \\ &= \text{P}(Z \geq 0.6708\ldots) \\ &= 1 - 0.7486 \\ &= 0.2514 \end{aligned}$$



(calc gives 0.25116…)
So accept awrt 0.251

c

$$\begin{aligned} \text{P}(43 < X \leq 52) &\approx \text{P}(43.5 \leq Y < 52.5) \\ &= \text{P}(-0.22\ldots \leq Z < 1.12\ldots) \\ &= 0.8686 - (1 - 0.5871) \\ &= 0.4557 \end{aligned}$$



(calc gives 0.45669…)
So accept awrt 0.456 ~ 0.457

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Exercise C, Question 3

Question:

The random variable $X \sim \text{Po}(60)$. Use a suitable approximation to estimate

a $\text{P}(X \leq 62)$,

b $\text{P}(X = 63)$,

c $\text{P}(55 \leq X < 65)$.

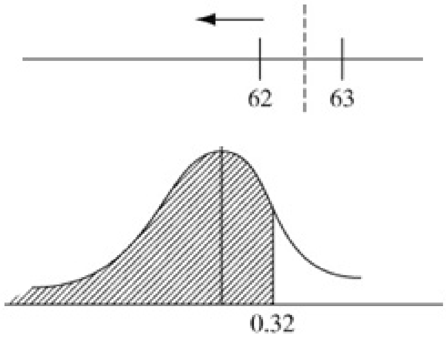
Solution:

$X \sim \text{Po}(60)$ $X \approx Y$

$Y \sim \text{N}(60, \sqrt{60}^2)$

a

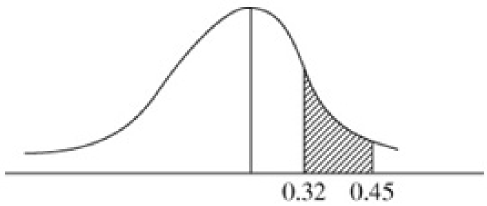
$$\begin{aligned} \text{P}(X \le 62) &\approx \text{P}(Y < 62.5) \\ &= \text{P}\left(Z < \frac{62.5 - 60}{\sqrt{60}}\right) \\ &= \text{P}(Z < 0.3227\ldots) \\ &= 0.6255 \end{aligned}$$



(calc gives 0.62655…)
So accept awrt 0.626 ~ 0.627

b

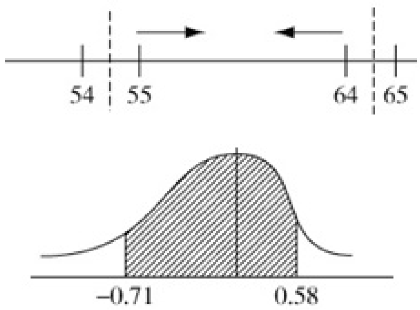
$$\begin{aligned} \text{P}(X = 63) &\approx \text{P}(62.5 \le Y < 63.5) \\ &= \text{P}(0.32\ldots \le Z < 0.45\ldots) \\ &= 0.6736 - 0.6255 \\ &= 0.0481 \end{aligned}$$



(calc gives 0.04775…)
So accept awrt 0.0480

c

$$\begin{aligned} \text{P}(55 \le X < 65) &= \text{P}(54.5 \le Y < 64.5) \\ &= \text{P}(-0.71\ldots \le Z < 0.58\ldots) \\ &= 0.7190 - (1 - 0.7611) \\ &= 0.4801 \end{aligned}$$



(Calc gives 0.48052…)
So accept awrt 0.480 ~ 0.481

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Exercise C, Question 4

Question:

The disintegration of a radioactive specimen is known to be at the rate of 14 counts per second. Using a normal approximation for a Poisson distribution, determine the probability that in any given second the counts will be

a 20, 21 or 22,

b greater than 10,

c above 12 but less than 16.

Solution:

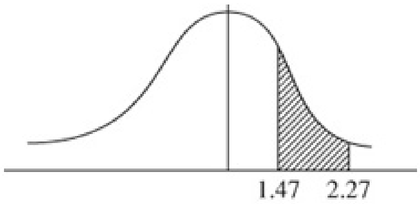
X = number of counts in one second

$X \sim \text{Po}(14)$

$Y \sim \text{N}(14, \sqrt{14}^2)$ $\qquad X \approx Y$

a

$$\begin{aligned} \text{P}(20 \le X \le 22) &\approx \text{P}(19.5 \le Y < 22.5) \\ &= \text{P}(1.47 \le Z < 2.27\ldots) \\ &= 0.9884 - 0.9292 \\ &= 0.0592 \end{aligned}$$

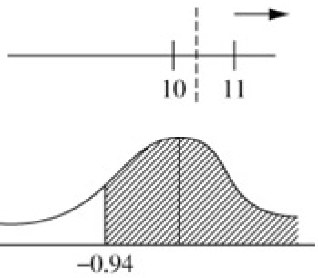


(calc gives 0.059237…)

So accept awrt 0.0590 ~ 0.0600

b

$$\begin{aligned} \text{P}(X > 10) &\approx \text{P}(Y \ge 10.5) \\ &= \text{P}(Z \ge -0.935\ldots) \\ &= 0.8264 \end{aligned}$$

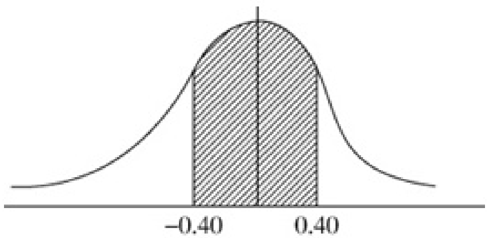


(calc gives 0.8252…)

So accept awrt 0.825 ~ 0.826

c

$$\begin{aligned} \text{P}(12 < X < 16) &\approx \text{P}(12.5 \le Y \le 15.5) \\ &= \text{P}(-0.40\ldots \le Z < 0.40\ldots) \\ &= 2 \times 0.1554 \\ &= 0.3108 \end{aligned}$$



(calc gives 0.311500…)

So accept awrt 0.311 ~ 0.312

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Exercise C, Question 5

Question:

A marina hires out boats on a daily basis. The mean number of boats hired per day is 15. Using the normal approximation for a Poisson distribution, find, for a period of 100 days

a how often 5 or fewer boats are hired,

b how often exactly 10 boats are hired,

c on how many days they will have to turn customers away if the marina owns 20 boats.

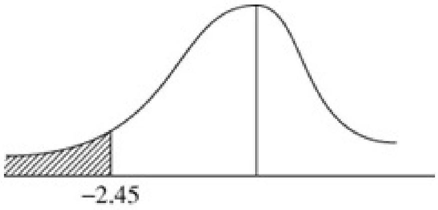
Solution:

X = number of boats hired per day
$X \sim \text{Po}(15)$

$Y \sim \text{N}(15, \sqrt{15}^2)$ $\quad X \approx Y$

a

$$\begin{aligned} \text{P}(X \le 5) &\approx \text{P}(Y < 5.5) \\ &= \text{P}(Z < -2.452\ldots) \\ &= 1 - 0.9929 \\ &= 0.0071 \end{aligned}$$

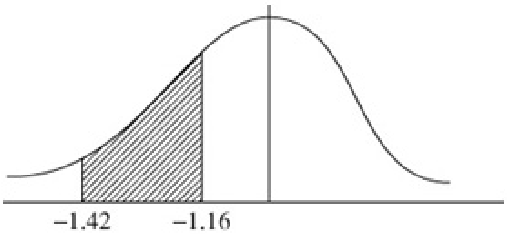


(calc gives 0.00708…)
So accept awrt 0.0070 to 0.0071

i.e. in 100 days $\approx$ 0.7 times
i.e. 1 day

b

$$\begin{aligned} \text{P}(X = 10) &\approx \text{P}(9.5 \le Y < 10.5) \\ &= \text{P}(-1.42 \le Z < -1.16) \\ &= [1 - 0.8770] - [1 - 0.9222] \\ &= 0.0452 \end{aligned}$$

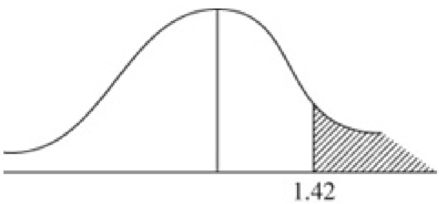


(calc gives 0.04484…)
So accept awrt 0.045

i.e. in 100 days $\approx$ 4.5 days
i.e. 4 or 5 days

c

$$\begin{aligned} \text{P}(X > 20) &\approx \text{P}(Y > 20.5) \\ &= \text{P}(Z > 1.42\ldots) \\ &= 1 - 0.9222 \\ &= 0.0778 \end{aligned}$$



(calc gives 0.07779…)
So accept 0.078

i.e. in 100 days $\approx$ 7.8 days
i.e. 8 days

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Exercise D, Question 1

Question:

A fair die is rolled and the number of sixes obtained is recorded.
Using suitable approximations, find the probability of
a no more than 10 sixes in 48 rolls of the die,
b at least 25 sixes in 120 rolls of the die.

Solution:

a $X =$ number of sixes in 48 rolls of a die

$X \sim B(48, \frac{1}{6})$

$Y \sim Po(8)$ $\mu = 8$

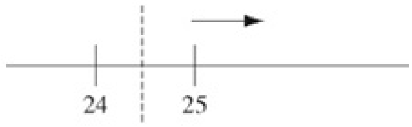
$$P(X \le 10) \approx P(Y \le 10) = 0.8159 \quad \text{(Poisson tables)}$$

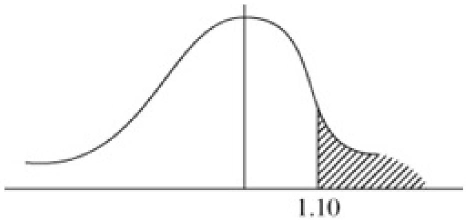
b $X =$ number of sixes in 120 rolls of a die

$X \sim B(120, \frac{1}{6})$

$Y \sim N(20, \sqrt{\frac{100}{6}}^2)$ or $Y \sim N(20, \sqrt{\frac{50}{3}}^2)$ $\mu = 20$

$$\begin{aligned} P(X \ge 25) &\approx P(Y \ge 24.5) \\ &= P(Z \ge 1.10\ldots) \\ &= 1 - 0.8643 \\ &= 0.1357 \end{aligned}$$





(calc gives 0.135172…)
So accept awrt 0.135 ~ 0.136

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Exercise D, Question 2

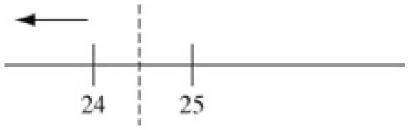
Question:

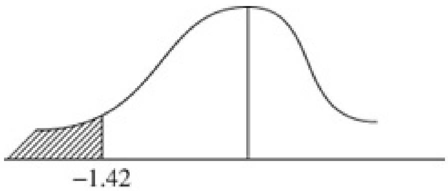
A fair coin is spun 60 times.
Use a suitable approximation to estimate the probability of obtaining fewer than 25 heads.

Solution:

$X =$ number of heads in 60 spins of a coin
$X \sim \mathrm{B}(60, 0.5)$
$Y \sim \mathrm{N}(30, \sqrt{15}^2)$ $\qquad X \approx Y$

$$\begin{aligned} \mathrm{P}(X < 25) &\approx \mathrm{P}(Y < 24.5) \\ &= \mathrm{P}(Z < -1.42\ldots) \\ &= 1 - 0.9222 \\ &= 0.0778 \end{aligned}$$





(calc gives 0.07779022…)
So accept awrt 0.0778

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Exercise D, Question 3

Question:

The owner of a local corner shop calculates that the probability of a customer buying a newspaper is 0.40 but the proportion of customers who spend over £10 is 0.04.
A random sample of 100 customer's shopping is recorded. Use suitable approximations to estimate the probability that in this sample

a at least half of the customers bought a newspaper,

b more than 5 of them spent over £10.

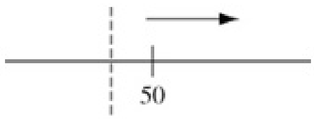
Solution:

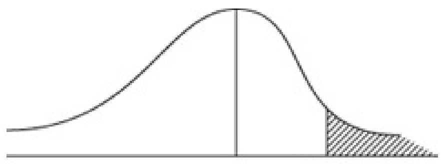
a $X =$ number of customers who bought a newspaper

$X \sim \mathrm{B}(100, 0.40)$

$Y \sim \mathrm{N}(40, \sqrt{24}^2)$ $\qquad X \approx Y$

$$\begin{aligned} \mathrm{P}(X \geq 50) &\approx \mathrm{P}(Y \geq 49.5) \\ &= \mathrm{P}\left(Z \geq \frac{49.5-40}{\sqrt{24}}\right) \\ &= \mathrm{P}(Z \geq 1.939\ldots) \\ &= 1-0.9738 \\ &= 0.0262 \end{aligned}$$





calc gives 0.026239…
So accept awrt 0.0262

b $T =$ number of customers who spent over £10

$T \sim \mathrm{B}(100, 0.04)$ $\qquad \mu = 4$

$S \approx \mathrm{Po}(4)$

$T \approx S$

$$\begin{aligned} \mathrm{P}(T > 5) &\approx \mathrm{P}(S \geq 6) && \text{(No continuity correction)} \\ &= 1-\mathrm{P}(S \leq 5) \\ &= 1-0.7851 && \text{(Poisson tables)} \\ &= 0.2149 \end{aligned}$$

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Exercise D, Question 4

Question:

Street light failures in a town occur at a rate of one every two days. Assuming that X, the number of street light failures per week, has a Poisson distribution, find the probabilities that the number of street lights that will fail in a given week is

a exactly 2,

b less than 6.

Using a suitable approximation estimate the probability that

c there will be fewer than 45 street light failures in a 10-week period.

Solution:

$X \sim \text{Po}(3.5)$ ← $\lambda = 7 \times \frac{1}{2}$

a

$$\begin{aligned} P(X=2) &= P(X \le 2) - P(X \le 1) \\ &= 0.3208 - 0.1359 \\ &= 0.1849 \end{aligned}$$

b

$$\begin{aligned} P(X<6) &= P(X \le 5) \\ &= 0.8576 \end{aligned}$$

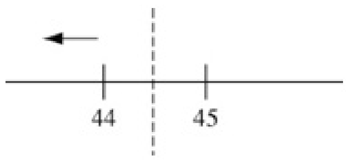
Let $Y =$ number of failures in 10-week period

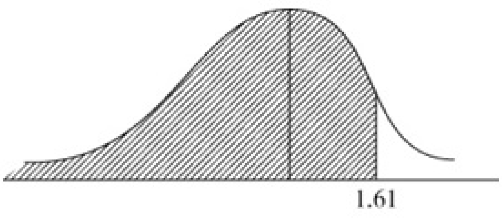
$Y \sim \text{Po}(35)$

$W \sim N(35, \sqrt{35}^2)$

c

$$\begin{aligned} P(Y<45) &\approx P(W \le 44.5) \\ &= P\left(Z \le \frac{44.5-35}{\sqrt{35}}\right) \\ &= P(Z \le 1.605\ldots) \\ &= 0.9463 \end{aligned}$$





calc gives 0.945840…
So accept awrt 0.946

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Exercise D, Question 5

Question:

Past records from a supermarket show that 20% of people who buy chocolate bars buy the family size bar. A random sample of 80 people is taken from those who had bought chocolate bars.

a Use a suitable approximation to estimate the probability that more than 20 of these 80 bought family size bars.

The probability of a customer buying a gigantic chocolate bar is 0.02.

b Using a suitable approximation estimate the probability that fewer than 5 customers in a sample of 150 buy a gigantic chocolate bar.

Solution:

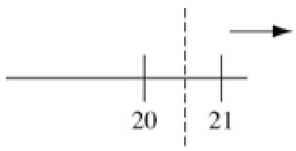
$X =$ number of people out of 80 who buy family size chocolate bars

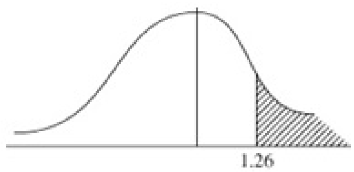
$X \sim \mathrm{B}(80, 0.20)$

$Y \sim \mathrm{N}(16, \sqrt{12.8}^2)$

a

$$\begin{aligned} \mathrm{P}(X > 20) &\approx \mathrm{P}(Y \geq 20.5) \\ &= \mathrm{P}\left(Z \geq \frac{20.5-16}{\sqrt{12.8}}\right) \\ &= \mathrm{P}(Z \geq 1.2577\ldots) \\ &= 1 - 0.8962 \\ &= 0.1038 \end{aligned}$$





calc gives 0.104234…

So accept awrt 0.104

b $G =$ number who buy a gigantic bar of chocolate

$G \sim \mathrm{B}(150, 0.02)$ ← $\mu = 150 \times 0.02 = 3$

$H \sim \mathrm{Po}(3)$ $G \approx H$

$$\begin{aligned} \mathrm{P}(G < 5) &\approx \mathrm{P}(H < 5) && \text{(No continuity correction)} \\ &= \mathrm{P}(H \leq 4) \\ &= 0.8153 && \text{(Poisson tables)} \end{aligned}$$

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Exercise A, Question 1

Question:

a Write down a brief description of a census.
b Write down two advantages of using a census rather than a sample.
c Write down two disadvantages of using a census rather than a sample.

Solution:

a A census is when **every member of a population** is used.
b ANY TWO FROM:
It is unbiased.
It gives an accurate, reliable answer.
It looks at every single member of the population.
c ANY TWO FROM:
It can take a long time to do.
It is often costly.
It is not easy to ensure that every member of the population is taken into account.

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Exercise A, Question 2

Question:

Write down which of the following are finite populations and which are infinite populations.

a Stars in the sky.

b Workers in a supermarket.

c The number of cows in Farmer Jacob's herd of cows.

Solution:

a is an infinite population.
b and **c** are finite populations.

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Exercise A, Question 3

Question:

a Write down a brief description of a sample.
b Write down one disadvantage of taking a sample rather than a census.
c Write down two advantages of taking a sample rather than a census.

Solution:

a EITHER: A sample is a subset of the population.
OR: A sample consists of a selected group of the members of the population.
b ANY ONE FROM:
It may be biased.
It may be subject to natural variation.
c ANY TWO FROM:
It is generally cheaper.
Data is often easier to get.
It generally takes less time.
It avoids testing to destruction.

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Exercise A, Question 4

Question:

A city council wants to know what people think about its recycling centre.
The council decides to carry out a sample survey to get the opinion of resident's views.

a Write down one reason why the council should not take a census.
b Suggest a suitable sampling frame.
c Identify the sampling units.

Solution:

a ANY ONE FROM:
It would be expensive.
It would be time consuming.
It would be difficult.
b A list of residents.
c A resident.

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Exercise A, Question 5

Question:

A factory manufactures climbing ropes. The manager of the factory decides to investigate the breaking point of the ropes.
Write down a reason, other than easier and cheaper, why he would not use a census.

Solution:

The climbing ropes would all be destroyed.

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Exercise A, Question 6

Question:

A supermarket manager wants to find out whether customers are satisfied with the range of products in the supermarket. He decides to do a survey.

a Write down a reason why the manager decides to use a sample rather than a census.

He decides to do a sample survey.

b Describe the sampling units for the sample survey.

c Give one advantage and one disadvantage of using a sample survey.

Solution:

a ANY ONE FROM:
It will be easier.
It will be quicker.
It will be cheaper.

b Customer.

c Advantages:
ANY ONE FROM:
It will be quick to do.
It will be easy to do.
It will not cost too much.
PLUS
Disadvantages:
ANY ONE FROM:
Not everyone's views will be known.
It might be biased.

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Exercise A, Question 7

Question:

A manager of a garage wants to know what his mechanics think about a new pension scheme designed for them. He decides to ask all the mechanics in the garage.

a Describe the population he will use.

b Write down the main advantage there will be in asking all his mechanics.

Solution:

a All the mechanics in the garage.

b Everyone's views will be known.

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Exercise A, Question 8

Question:

Each computer produced by a manufacturer is stamped with a unique serial number. ITPro Limited make their computers in batches of 1000. Before selling the computers, they test a random sample of 5 to see what electrical overload they will take before breaking down.

a Give one reason, other than to save time and cost, why a sample is taken rather than a census.

b Suggest a suitable sampling frame.

c Identify the sampling units.

Solution:

a If a census were used all the computers would be destroyed.

b The list of unique serial numbers.

c A computer.

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Exercise B, Question 1

Question:

A forester wants to estimate the height of the trees in a forest. He measures the heights of 50 randomly selected trees and works out the mean height. State with a reason whether or not this mean is a statistic.

Solution:

This mean is from the values of a sample so it is a statistic.

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Exercise B, Question 2

Question:

A random sample $M_1, M_2, M_3, \ldots, M_n$ is taken from a population with unknown mean μ. For each of the following state whether or not it is a statistic.

a $\dfrac{M_3 + M_8}{2}$

b $\dfrac{\Sigma M}{n}$

c $\dfrac{\Sigma M}{n} - \mu^2$

Solution:

i) and ii) are statistics.
iii) is not a statistic since it uses μ.

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Exercise B, Question 3

Question:

The owners of a chain of hairdressing shops want to introduce the use of overalls in all the shops. The random variable Y is defined as

$Y = 0$ if the staff are happy to wear the overalls and

$Y = 1$ if the staff are unhappy about wearing the overalls.

a Suggest a suitable population and identify any parameter of interest.

A random sample of 20 of the hairdressers are asked whether they are happy or unhappy about wearing the overalls.

b Write down the name of the sampling distribution of the statistic $X = \sum_{1}^{20} Y$.

Solution:

a All the hairdressers who work for the chain of hairdressing shops.
The proportion p of the staff happy to wear overalls.

b This is a binomial distribution since we are only interested in two options – whether or not the hairdressers are happy to wear the overalls.

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Exercise B, Question 4

Question:

A secretary makes spelling mistakes at the rate of 5 for every 10 pages. He has just finished typing a six-page document.

a Write down a suitable sampling distribution for the number of spelling mistakes in his document.

b Find the probability that there has been fewer than 2 spelling mistakes in the document.

Solution:

a) Po(3)

b) $P(X < 2) = P(X \leq 1) = 0.1991$

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Exercise B, Question 5

Question:

A bag contains a large number of coins. 50% are 50 pence coins.
25% are 20 pence coins. 25% are 10 pence coins.

a Find the mean, μ, and the variance, σ, for the value of this population of coins.

A random sample of 2 coins is chosen from the bag.

b List all the possible samples that can be chosen.

c Find the sampling distribution for the mean.

$$\overline{X} = \frac{X_1 + X_2}{2}$$

Solution:

a

X	50	20	10
X^2	2500	400	100
p	0.5	0.25	0.25

Mean $= (50 \times 0.5) + (20 \times 0.25) + (10 \times 0.25) = 25 + 5 + 2.5 = \mathbf{32.5}$

Variance $= ((2500 \times 0.5) + (400 \times 0.25) + (100 \times 0.25)) - 32.5^2$

$= (1250 + 100 + 25) - 1056.25 = 1375 - 1056.25 = 318.75$

b (50, 50)
(50, 20) (20, 50)
(50, 10) (10, 50)
(20, 20)
(20, 10) (10, 20)
(10, 10)

c $P(\overline{X} = 50) = 0.5 \times 0.5 = 0.25$

$P(\overline{X} = 35) = (0.5 \times 0.25) \times 2 = 0.25$

$P(\overline{X} = 30) = (0.5 \times 0.25) \times 2 = 0.25$

$P(\overline{X} = 20) = (0.25 \times 0.25) = 0.0625$

$P(\overline{X} = 15) = (0.25 \times 0.25) \times 2 = 0.125$

$P(\overline{X} = 10) = (0.25 \times 0.25) = 0.0625$

So the sampling distribution for the mean is:

$\overline{X}$	50	35	30	20	15	10
$P(\overline{X})$	0.25	0.25	0.25	0.0625	0.125	0.0625

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Exercise B, Question 6

Question:

A manufacturer makes three sizes of toaster. 40% of the toasters sell for £16, 50% sell for £20 and 10% sell for £30.

a Find the mean and variance of the value of the toasters.

A sample of 2 toasters is sent to a shop.

b List all the possible prices of the samples that could be sent.

c Find the sampling distribution for the mean price $\bar{X}$ of these samples.

Solution:

a

X	16	20	30
X^2	256	400	900
p	0.4	0.5	0.1

$$\text{Mean} = (16\times 0.4) + (20\times 0.5) + (30\times 0.1) = 6.4 + 10 + 3 = 19.4$$

$$\text{Variance} = ((256\times 0.4) + (400\times 0.5) + (900\times 0.1)) - 19.4^2$$
$$= (102.4 + 200 + 90) - 376.36 = 392.4 - 376.36 = 16.04$$

b

(16, 16)
(16, 20) (20, 16)
(16, 30) (30, 16)
(30, 30)
(30, 20) (20, 30)
(20, 20)

c

$P(\bar{X} = 16) = 0.4\times 0.4 = 0.16$
$P(\bar{X} = 18) = (0.4\times 0.5)\times 2 = 0.4$
$P(\bar{X} = 23) = (0.4\times 0.1)\times 2 = 0.08$
$P(\bar{X} = 30) = (0.1\times 0.1) = 0.01$
$P(\bar{X} = 25) = (0.1\times 0.5)\times 2 = 0.1$
$P(\bar{X} = 20) = (0.5\times 0.5) = 0.25$

So the sampling distribution for the mean is: [use UC X throughout]

$\bar{X}$	16	18	20	23	25	30
$P(\bar{X})$	0.16	0.4	0.25	0.08	0.1	0.01

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Exercise B, Question 7

Question:

A supermarket sells a large number of 3-litre and 2-litre cartons of milk. They are sold in the ratio 3:2

a Find the mean and variance of the milk content in this population of cartons.

A random sample of 3 cartons is taken from the shelves (X_1, X_2 and X_3).

b List all the possible samples.

c Find the sampling distribution of the mean $\bar{X}$.

d Find the sampling distribution of the mode M.

e Find the sampling distribution of the median N of these samples.

Solution:

a

X	3	2
X^2	9	4
p	0.6	0.4

$$\text{Mean} = (3\times 0.6) + (2\times 0.4) = 1.8 + 0.8 = 2.6$$
$$\text{Variance} = ((9\times 0.6) + (4\times 0.4)) - 2.6^2 = (5.4 + 1.6) - 6.76 = 0.24$$

b (3, 3, 3)
(3, 3, 2) (3, 2, 3) (2, 3, 3)
(3, 2, 2) (2, 3, 2) (2, 2, 3)
(2, 2, 2)

c

$\mathrm{P}(\bar{X} = 3) = 0.6^3 = 0.216$

$\mathrm{P}(\bar{X} = 2\frac{2}{3}) = (0.6\times 0.6\times 0.4)\times 3 = 0.432$

$\mathrm{P}(\bar{X} = 2\frac{1}{3}) = (0.6\times 0.4\times 0.4)\times 3 = 0.288$

$\mathrm{P}(\bar{X} = 2) = 0.4^3 = 0.064$

So the sampling distribution for $\bar{X}$ is:

$\bar{X}$	3	$2\frac{2}{3}$	$2\frac{1}{3}$	2
$\mathrm{P}(\bar{X})$	0.216	0.432	0.288	0.064

d The mode can be 3 or 2
$\mathrm{P}(M = 3) = 0.216 + 0.432 = 0.648$
$\mathrm{P}(M = 2) = 0.288 + 0.064 = 0.352$
So the sampling distribution for the mode M is:

M	3	2
$\mathrm{P}(M)$	0.648	0.352

e

The median can be 3 (i.e. the cases (3, 3, 3) (3, 3, 2) (3, 2, 3) (2, 3, 3))
or 2 (i.e. the cases (3, 2, 2) (2, 3, 2) (2, 2, 3) (2, 2, 2))
$\mathrm{P}(N = 3) = 0.6^3 + 3(0.6\times 0.6\times 0.4) = 0.216 + 0.432 = 0.648$
$\mathrm{P}(N = 2) = 0.4^3 + 3(0.6\times 0.4\times 0.4) = 0.064 + 0.288 = 0.352$
So the sampling distribution for the median N is:

N	3	2
$\mathrm{P}(N)$	0.648	0.352

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Exercise C, Question 1

Question:

A doctor's surgery is to offer health checks to all its patients over 65. In order to estimate the amount of time needed to do these health checks the doctor decides to do the health check for a random sample of 20 patients over 65.

a Write down a suitable sampling frame that the doctor might use.

b Identify the sampling units.

Solution:

a A list of all the patients on the surgery books.

b A patient.

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Exercise C, Question 2

Question:

The owners of a large gym wish to change the opening hours. They want to find out whether the members will be happy with the new hours. They ask a random sample of 30 members.

a Write two likely reasons why the owners did not ask all the members.

b Suggest a suitable sampling frame.

c Identify the sampling units.

Solution:

a ANY TWO FROM:
It would take too long.
It could cost too much.
It could be difficult to get hold of all members.

b
A list of all members of the gym.

c
A member of the gym.

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Exercise C, Question 3

Question:

a Write down a reason why a sampling frame and a population may not be the same.

b Explain briefly why a sample is often used rather than a census.

Solution:

a A sampling frame has to be some sort of list – it may not be possible to list a population.

b A sample is usually easier to do, quicker to do and not as costly as a census.

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Exercise C, Question 4

Question:

a Explain what a statistic is.

A random sample $Y_1, Y_2, \ldots, Y_n$ is taken from a population with unknown mean μ.

b For each of the following state with a reason whether or not it is a statistic.

i $\frac{Y_1 + Y_2 + Y_3}{4}$

ii $\frac{\Sigma Y}{n} - \mu$

Solution:

a A statistic is a quantity calculated solely from the observations of a sample.

b i) is a statistic ii) is not a statistic as it depends on the value μ.

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Exercise C, Question 5

Question:

A company manufactures electric light bulbs. They wish to see how many hours the light bulbs will work before failing. The company decides to test every 200th light bulb coming off the assembly line.

a Write down why the company does not test every light bulb.

b Identify the sampling units.

Solution:

a The light bulbs would all be destroyed.

b A light bulb.

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Exercise C, Question 6

Question:

A call centre has 400 people operating the telephones. The manager decides that he needs to know how long the operatives are spending on each call. He times a random sample of 30 operators over one day and works out the mean time per call.

a Write down two advantages of using a sample rather than a census in this case.

b Write down one disadvantage of using a sample in this case.

A sample is to be taken.

c Suggest a sampling frame.

d Identify the sampling units

e Is the mean time the manager works out from the sample a statistic? Give a reason for your answer.

Solution:

a ANY TWO FROM:
It is quicker to do.
It is cheaper to do.
It is easier to do.

b It can be biased. OR it is subject to natural variations.

c A numbered list of all 400 call-centre operatives.

d A call-centre operative.

e Yes, because he is using only the values from a sample. There are no parameters.

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Exercise C, Question 7

Question:

A flower shop has ten florists. The owner wants to know whether the florists are happy with the quality of the flowers being delivered to the shop. The owner asks all the florists their views. Write down two reasons why the owner of the florist shop uses a census.

Solution:

ANY TWO FROM:
It takes into account everyone's views.
It is unbiased
To take a sample when the population is only 10 would be silly.

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Exercise C, Question 8

Question:

The weights of tomatoes in a greenhouse are assumed to have mean μ and standard deviation σ.
A sample of 20 tomatoes were each weighed and their weights were recorded. If the sample is represented by $X_1, X_2, \ldots, X_{20}$ state whether or not the following are statistics.

a $\frac{X_1 + X_{20}}{3}$

b $\frac{\Sigma X}{20}$

c $\Sigma X^2 + \mu$

d $\frac{\Sigma X^2}{20} - \sigma^2$

Solution:

a) and b) are statistics c) and d) are not statistics since they involve a population parameter.

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Exercise C, Question 9

Question:

A large box of coins contains 5p, 10p, and 20p coins in the ratio 3:2:1.

a Find the mean μ and the variance σ^2 of the value of the coins.

A random sample of 2 coins is taken from the box and their values Y_1 and Y_2 are recorded.

b List all the possible samples that can be taken.

c Find the sampling distribution for the mean $(\bar{Y})$.

Solution:

a

$$\text{Mean} = (5\times\frac{3}{6}) + (10\times\frac{2}{6}) + (20\times\frac{1}{6}) = \frac{15}{6} + \frac{20}{6} + \frac{20}{6} = \frac{55}{6} = 9\frac{1}{6}$$

$$\text{Variance} = (25\times\frac{3}{6}) + (100\times\frac{2}{6}) + (400\times\frac{1}{6}) - (9\frac{1}{6})^2$$

$$= \frac{75}{6} + \frac{200}{6} + \frac{400}{6} - \frac{3025}{36} = \frac{675}{6} - \frac{3025}{36} = 28.47$$

b (5, 5) (10, 10) (20, 20)
(5, 10) (10, 5) (5, 20) (20, 5)
(10, 20) (20, 10)

c Possible means are:
5, 10, 20,
7.5, 12.5, 15 These all occur twice

$P(\bar{Y} = 5) = \frac{3}{6}\times\frac{3}{6} = \frac{9}{36} = \frac{1}{4}$

$P(\bar{Y} = 10) = \frac{2}{6}\times\frac{2}{6} = \frac{4}{36} = \frac{1}{9}$

$P(\bar{Y} = 20) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

$P(\bar{Y} = 7.5) = \frac{3}{6}\times\frac{2}{6}\times 2 = \frac{12}{36} = \frac{1}{3}$

$P(\bar{Y} = 12.5) = \frac{3}{6}\times\frac{1}{6}\times 2 = \frac{6}{36} = \frac{1}{6}$

$P(\bar{Y} = 15) = \frac{2}{6}\times\frac{1}{6}\times 2 = \frac{4}{36} = \frac{1}{9}$

So the sampling distribution for the means is:

$\bar{Y}$	5	7.5	10	12.5	15	20
$P(\bar{Y})$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{36}$

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Exercise C, Question 10

Question:

A bag contains a large number of counters

60% have a value of 6

40% have a value of 10.

A random sample of 3 counters is drawn from the bag.

a Write down all the possible samples.

b Find the sampling distribution for the median N.

c Find the sampling distribution for the mode M.

Solution:

a (6, 6, 6)

(6, 6, 10) (6, 10, 6) (10, 6, 6)

(6, 10, 10) (10, 6, 10) (10, 10, 6)

(10, 10, 10)

b Medians can be 6 or 10. If put in order 4 give a median of 10 and 4 give a median of 6

$$P(N=6)=\left(\frac{6}{10}\times\frac{6}{10}\times\frac{6}{10}\right)+3\left(\frac{6}{10}\times\frac{6}{10}\times\frac{4}{10}\right)=\frac{216}{1000}+\frac{432}{1000}=\frac{648}{1000}=0.648$$

$$P(N=10)=3\left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{4}{10}\times\frac{4}{10}\right)=\frac{288}{1000}+\frac{64}{1000}=\frac{352}{1000}=0.352$$

So distribution of median is:

N	6	10
P(M)	0.648	0.352

c The mode is either 6 or 10

$$P(M=6)=\left(\frac{6}{10}\times\frac{6}{10}\times\frac{6}{10}\right)+3\left(\frac{6}{10}\times\frac{6}{10}\times\frac{4}{10}\right)=\frac{216}{1000}+\frac{432}{1000}=\frac{648}{1000}=0.648$$

$$P(M=10)=3\left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{4}{10}\times\frac{4}{10}\right)=\frac{288}{1000}+\frac{64}{1000}=\frac{352}{1000}=0.352$$

So distribution of mode is:

M	6	10
P(M)	0.648	0.352

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Exercise A, Question 1

Question:

a Describe what is meant by the expression 'a statistical hypothesis'.

b Describe the difference between the null hypothesis and the alternative hypothesis.

c What symbols do we use to denote the null and alternative hypotheses?

Solution:

a This is an assumption made about a population parameter that we test using evidence from a sample.

b The null hypothesis is what we assume to be correct and the alternative hypothesis is what we conclude if our assumption is wrong.

c Null Hypothesis = H_0 Alternative hypothesis = H_1

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Exercise A, Question 2

Question:

Dmitri wants to see whether a die is biased towards the value 6.
He throws the die 60 times and counts the number of sixes he gets.

a Describe the test statistic.

b Write down a suitable null hypothesis to test this die.

c Write down a suitable alternative hypothesis to test this die.

Solution:

a The test statistic is N – the number of sixes.

b $H_0 : p = \frac{1}{6}$

c $H_1 : p > \frac{1}{6}$

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Exercise A, Question 3

Question:

Shell wants to test to see whether a coin is biased. She tosses the coin 100 times and counts the number of times she gets a head.

a Describe the test statistic.

b Write down a suitable null hypothesis to test this coin.

c Write down a suitable alternative hypothesis to test this coin.

Solution:

a The test statistic is N – the number of times you get a head.

b $H_0: p = \frac{1}{2}$

c $H_1: p \neq \frac{1}{2}$

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Exercise A, Question 4

Question:

Over a long period of time it is found that the mean number of accidents, λ, occurring at a particular crossroads is 4 per month. New traffic lights are installed. Jess decides to test to see whether the proportion of accidents has increased, decreased or changed in any way.

a Describe the test statistic.

b Write down a suitable null hypothesis to test Jess' theory.

c Write down three possible alternative hypotheses to test Jess' theory.

Solution:

a The test statistic is the number of accidents (in a given month or other specified time period).

b H_0: $\lambda = 4$

c Change H_1: $\lambda \neq 4$ (2 tail); or Decrease H_1: $\lambda < 4$ or Increase H_1: $\lambda > 4$ (both one tail).

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Exercise A, Question 5

Question:

In a survey it was found that 4 out of 10 people supported a certain particular political party. Chang wishes to test whether or not there has been a change in the proportion (p) of people supporting the party.

a Write down whether it would be best to use a one-tail test or a two-tail test. Give a reason for your answer.

b Suggest suitable hypotheses.

Solution:

a A two tail test would be best. The support could get better or could get worse.

b $H_0: p = 0.4$

$H_1: p \neq 0.4$

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Exercise A, Question 6

Question:

In a manufacturing process the proportion (p) of faulty articles has been found, from long experience, to be 0.1.
The proportion of faulty articles in the first batch produced by a new process is measured.
The proportion of faulty articles in this batch is 0.09.
The manufacturers wish to test at the 5% level of significance whether or not there has been a reduction in the proportion of faulty articles.

a Suggest a suitable test statistic.
b Suggest suitable hypotheses.
c Explain the condition under which the null hypothesis is rejected.

Solution:

a A suitable test statistic is p – the proportion of faulty articles in a batch.
b $H_0: p = 0.1$ $H_1: p < 0.1$
c If the probability of the proportion being 0.09 or less is 5% or less the null hypothesis is rejected.

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Exercise A, Question 7

Question:

A spinner has 4 sides numbered 1, 2, 3 and 4. Hajdra thinks it is biased to give a one when spun. She spins 5 times and counts the number of times, M, that she gets a 1.

a Describe the test statistics M.

She decides to do a test with a level of significance of 5%.

b What values of M would cause the null hypothesis to be rejected.

Solution:

a The test statistic is M – the number of times Hajdra gets a 1.

b B (5, 0.25)

N	0	1	2	3	4	5
P($N = n$)	0.237	0.395	0.264	0.088	0.015	0.001

There is a $0.015 + 0.001 = 0.016 = 1.6\%$ chance of getting one 4 or 5 times.
There is a $0.088 + 0.001 + 0.015 = 0.104 = 10.4\%$ chance of getting one 3, 4 or 5 times.
If N is 4 or 5 then the null hypothesis would be rejected, since P(4 or more) $= 1.6\% < 5\%$.

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Exercise B, Question 1

Question:

For each of the questions 1 to 7 carry out the following tests using the binomial distribution where the random variable, X, represents the number of successes.

$H_0: p = 0.25$; $H_1: p > 0.25$; $n = 10, x = 5$ and using a 5% level of significance.

Solution:

Distribution B(10, 0.25)

$H_0: p = 0.25 \quad H_1: p > 0.25$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.9219$

$= 0.0781$

$0.0781 > 0.05$

There is insufficient evidence to reject H_0.

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Exercise B, Question 2

Question:

$H_0 : p = 0.40$; $H_1 : p < 0.40$; $n = 10, x = 1$ and using a 5% level of significance.

Solution:

Distribution B(10, 0.40)
$H_0 : p = 0.40 \quad H_1 : p < 0.40$
$P(X \leq 1) = 0.0464$
$0.0464 < 0.05$
There is sufficient evidence to reject H_0 so $p < 0.04$.

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Exercise B, Question 3

Question:

$H_0: p = 0.30$; $H_1: p > 0.30$; $n = 20, x = 10$ and using a 5% level of significance.

Solution:

Distribution B(20, 0.30)

$H_0: p = 0.30 \quad H_1: p > 0.30$

$$\begin{aligned} P(X \geq 10) &= 1 - P(X \leq 9) \\ &= 1 - 0.9520 \\ &= 0.0480 \end{aligned}$$

$0.0480 < 0.05$

There is sufficient evidence to reject H_0 so $p > 0.30$.

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Exercise B, Question 4

Question:

$H_0: p = 0.45$; $H_1: p < 0.45$; $n = 20, x = 3$ and using a 1% level of significance.

Solution:

Distribution B(20, 0.45)
$H_0: p = 0.45$ $H_1: p < 0.45$
$P(X \leq 3) = 0.0049$
$0.0049 < 0.01$
There is sufficient evidence to reject H_0 so $p < 0.45$.

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Exercise B, Question 5

Question:

$H_0 : p = 0.50$; $H_1 : p \neq 0.50$; $n = 30, x = 10$ and using a 5% level of significance.

Solution:

Distribution B(30, 0.50)
$H_0 : p = 0.50$ $H_1 : p \neq 0.50$
$P(X \leq 10) = 0.0494$
$0.0494 > 0.025$ (two-tailed)
There is insufficient evidence to reject H_0
so there is no reason to doubt $p = 0.5$.

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Exercise B, Question 6

Question:

$H_0 : p = 0.28$; $H_1 : p < 0.28$; $n = 20, x = 2$ and using a 5% level of significance.

Solution:

Distribution B(20, 0.28)
$H_0 : p = 0.28 \quad H_1 : p < 0.28$
$$\begin{aligned}P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2)\\ &= 0.72^{20} + 20 \times 0.72^{19} \times 0.28 + 190 \times 0.72^{18} \times 0.28^2\\ &= 0.0014 + 0.0109 + 0.0403\\ &= 0.0526\end{aligned}$$
$0.0526 > 0.05$
There is insufficient evidence to reject H_0
so there is no reason to doubt $p = 0.28$.

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Exercise B, Question 7

Question:

$H_0: p = 0.32$; $H_1: p > 0.32$; $n = 8, x = 7$ and using a 5% level of significance.

Solution:

Distribution B(8, 0.32)
$H_0: p = 0.32 \quad H_1: p > 0.32$
$P(X \geq 7) = P(X = 7) + P(X = 8)$
$= 8 \times 0.32^7 \times 0.68 + 0.32^8$
$= 0.0019 + 0.0001$
$= 0.0020$
$0.0020 < 0.05$
There is sufficient evidence to reject H_0 so $p > 0.32$.

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Exercise B, Question 8

Question:

For each of the questions 8 to 10 carry out the following tests using the Poisson distribution where λ represents its mean.

$H_0 : \lambda = 8$; $H_1 : \lambda < 8$; $x = 3$ and using a 5% level of significance.

Solution:

Distribution Po(8)
$H_0 : \lambda = 8 \quad H_1 : \lambda < 8$
$P(X \leq 3) = 0.0424$
$0.0424 < 0.05$
There is sufficient evidence to reject H_0 so $\lambda < 8$.

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Exercise B, Question 9

Question:

$H_0 : \lambda = 6.5$; $H_1 : \lambda < 6.5$; $x = 2$ and using a 1% level of significance.

Solution:

Distribution Po(6.5)
$H_0 : \lambda = 6.5 \quad H_1 : \lambda < 6.5$
$P(X \leq 2) = 0.0430$
$0.0430 > 0.01$ (1% sig. level)
There is insufficient evidence to reject H_0 so there is no reason to doubt $\lambda < 6.5$

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Exercise B, Question 10

Question:

$H_0 : \lambda = 5.5$; $H_1 : \lambda > 5.5$; $x = 8$ and using a 5% level of significance.

Solution:

Distribution Po(5.5)
$H_0 : \lambda = 5.5 \quad H_1 : \lambda > 5.5$
$P(X \geq 8) = 1 - P(X \leq 7)$
$= 1 - 0.8095$
$= 0.1905$
$0.1905 > 0.05$
There is insufficient evidence to reject H_0 so there is no reason to doubt $\lambda > 5.5$

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Exercise B, Question 11

Question:

The manufacturer of 'Supergold' margarine claims that people prefer this to butter. As part of an advertising campaign he asked 5 people to taste a sample of 'Supergold' and a sample of butter and say which they prefer. Four people chose 'Supergold'. Assess the manufacturer's claim in the light of this evidence. Use a 5% level of significance.

Solution:

Distribution B(5, 0.5)
$H_0 : p = 0.5 \quad H_1 : p > 0.5$
$P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.8125$
$= 0.1875$
$0.1875 > 0.05$
There is insufficient evidence to reject H_0 (not significant).
There is insufficient evidence to suggest
that people prefer 'Supergold' to butter.

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Exercise B, Question 12

Question:

I tossed a coin 20 times and obtained a head on 6 occasions. Is there evidence that the coin is biased? Use a 5% two-tailed test.

Solution:

Distribution B(20, 0.50)
$H_0 : p = 0.50$ $H_1 : p \neq 0.50$
$P(X \leq 6) = 0.0577$
$0.0577 > 0.025$ (two tailed)
There is insufficient evidence to reject H_0 so we conclude there is no evidence the coin is biased.

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Exercise B, Question 13

Question:

A die used in playing a board game is suspected of not giving the number 6 often enough. During a particular game it was rolled 12 times and only one 6 appeared. Does this represent significant evidence, at the 5% level of significance, that the probability of a 6 on this die is less than $\frac{1}{6}$?

Solution:

Distribution $B(12, \frac{1}{6})$

$H_0 : p = \frac{1}{6}$ $H_1 : p < \frac{1}{6}$

$P(X \leq 1) = P(X = 0) + P(X = 1)$

$= \frac{5}{6}^{12} + 12\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)$

$= 0.3813$

$0.3813 > 0.05$

There is insufficient evidence to reject H_0 (not significant).

There is no evidence that the probability is less than $\frac{1}{6}$.

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Exercise B, Question 14

Question:

The success rate of the standard treatment for patients suffering from a particular skin disease is claimed to be 68%.

a In a sample of n patients, X is the number for which the treatment is successful. Write down a suitable distribution to model X. Give reasons for your choice of model.

A random sample of 10 patients receives the standard treatment and in only 3 cases was the treatment successful. It is thought that the standard treatment was not as effective as it is claimed.

b Test the claim at the 5% level of significance.

Solution:

a

Distribution $B(n, 0.68)$

Fixed number of trials.

Outcomes of trials are independent.

There are two outcomes success and failure.

The probability of success is constant.

b

Distribution $B(10, 0.68)$

$H_0 : p = 0.68 \quad H_1 : p < 0.68$

$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= 0.32^{10} + 10(0.32)^9(0.68) + 45(0.32)^8(0.68)^2 + 120(0.32)^7(0.68)^3$

$= 0.0000 + 0.0002 + 0.0023 + 0.0130$

$= 0.0155$

$0.0155 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.68$.

The treatment is not as effective as is claimed.

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Exercise B, Question 15

Question:

Every year a statistics teacher takes her class out to observe the traffic passing the school gates during a Tuesday lunch hour. Over the years she has established that the average number of lorries passing the gates in a lunch hour is 7.5. During the last 12 months a new bypass has been built and the number of lorries passing the school gates in this year's experiment was 4. Test, at the 5% level of significance, whether or not the mean number of lorries passing the gates during a Tuesday lunch hour has been reduced.

Solution:

Distribution Po(7.5)
$H_0 : \lambda = 7.5 \quad H_1 : \lambda < 7.5$
$P(X \leq 4) = 0.1321$
$0.1321 > 0.05$
There is insufficient evidence to reject H_0 (not significant).
There is no evidence of a decrease in the number of lorries passing the gates in a lunch hour.

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Exercise B, Question 16

Question:

Over a long period, John has found that the bus taking him to school arrives late on average 9 times per month. In the month following the start of the new summer schedule the bus arrives late 13 times. Assuming that the number of times the bus is late has a Poisson distribution, test, at the 5% level of significance, whether the new schedules have in fact increased the number of times on which the bus is late. State clearly your null and alternative hypotheses.

Solution:

Distribution Po(9)

$H_0 : \lambda = 9 \quad H_1 : \lambda > 9$

$P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - 0.8758$

$= 0.1242$

$0.1242 > 0.05$

There is insufficient evidence to reject H_0 (not significant).

There is no evidence that the new schedules have increased the number of times the bus is late.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

For each of the questions 1 to 6 find the critical region for the test statistic X representing the number of successes. Assume a binomial distribution.

$H_0: p = 0.20$; $H_1: p > 0.20$; $n = 10$, using a 5% level of significance.

Solution:

B(10, 0.2)
$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.8791 = 0.1209 > 0.05$
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9672 = 0.0328 < 0.05$
The critical value is $x = 5$ and the critical region is $X \geq 5$
since $P(X \geq 5) = 0.0328 < 0.05$.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

$H_0: p = 0.15$; $H_1: p < 0.15$; $n = 20$, using a 5% level of significance.

Solution:

B(20, 0.15)
$P(X \leq 1) = 0.1756 > 0.05$
$P(X = 0) = 0.0388 < 0.05$
The critical value is $x = 0$ and the critical region is $X = 0$.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

$H_0 : p = 0.40$; $H_1 : p \neq 0.40$; $n = 20$, using a 5% level of significance (2.5% at each tail).

Solution:

B(20, 0.4)
$P(X \leq 4) = 0.0510 > 0.025$
$P(X \leq 3) = 0.0160 < 0.025$
The critical value is $x = 3$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.025$
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.025$
The critical value is $x = 13$
The critical region is $X \geq 13$ and $X \leq 3$.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

$H_0 : p = 0.18$; $H_1 : p < 0.18$; $n = 20$, using a 1% level of significance.

Solution:

B(20, 0.18)
$P(X = 0) = 0.82^{20} = 0.0189 < 0.05$
$P(X \leq 1) = 0.0189 + 20(0.82)^{19}(0.18) = 0.0189 + 0.0829 = 0.1018 > 0.05$
The critical value is $x = 0$. The critical region is $X = 0$.

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Exercise C, Question 5

Question:

$H_0 : p = 0.63$; $H_1 : p > 0.36$; $n = 10$, using a 5% level of significance.

Solution:

B(10, 0.63)
$P(X = 10) = 0.63^{10} = 0.0098 < 0.05$
$P(X \geqslant 9) = 0.0098 + 10(0.63)^9\,(0.37) = 0.0675 > 0.05$
The critical value is $x = 10$ and the critical region is $X = 10$.

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Exercise C, Question 6

Question:

$H_0 : p = 0.22$; $H_1 : p \neq 0.22$; $n = 10$, using a 1% level of significance (0.005 at each tail).

Solution:

$B(10, 0.22)$
$P(X = 0) = 0.78^{10} = 0.0834 > 0.005$
$P(X \geq 6) = 0.010 > 0.005$
$P(X \geq 7) = 0.0016 < 0.005$
Critical region is $X \geq 7$.

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Exercise C, Question 7

Question:

For each of the questions 7 to 9 find the critical region for the test statistic X given that X has a $\text{Po}(\lambda)$ distribution.

$H_0: \lambda = 4$; $H_1: \lambda > 4$; using a 5% level of significance.

Solution:

Po(4)
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9489 = 0.0511 > 0.05$
$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214 < 0.05$
Critical region is $X \geq 9$.

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Exercise C, Question 8

Question:

$H_0 : \lambda = 9$; $H_1 : \lambda < 9$; using a 1% level of significance.

Solution:

Po(9)
$P(X \leq 2) = 0.0062 < 0.01$
$P(X \leq 3) = 0.0212 > 0.01$
Critical region is $X \leq 2$.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

$H_0 : \lambda = 3.5$; $H_1 : \lambda < 3.5$; using a 5% level of significance.

Solution:

Po(3.5)
$P(X = 0) = 0.0302 < 0.05$
$P(X \le 1) = 0.1359 > 0.05$
Critical region is $X = 0$.

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Exercise C, Question 10

Question:

A seed merchant usually kept her stock in carefully monitored conditions. After the Christmas holidays one year she discovered that the monitoring system had broken down and there was a danger that the seed might have been damaged by frost. She decided to check a sample of 10 seeds to see if the proportion p that germinates had been reduced from the usual value of 0.85. Find the critical region for a one-tailed test using a 5% level of significance.

Solution:

$X \sim B(10, 0.85)$ let $Y \sim B(10, 0.15)$
$H_0 : p = 0.85 \quad H_1 : p < 0.85$
$P(X \leq 6) = P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - 0.9500 = 0.0500 = 0.05$
Critical region is $X \leq 6$.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 11

Question:

The national proportion of people experiencing complications after having a particular operation in hospitals is 20%. A particular hospital decides to take a sample of size 20 from their records.

a State all the possible numbers of patients with complications that would cause them to decide that their proportion of complications differs from the national figure at the 5% level of significance ensuring that the probability in each tail is as near to 2.5% as possible.

The hospital finds that out of 20 such operations, 8 of their patients experienced complications.

b Find critical regions, at the 5% level of significance, to test whether or not their proportion of complications differs from the national proportion. The probability in each tail should be as near 2.5% as possible.

c State the actual significance level of the above test.

Solution:

$X \sim B(20, 0.20)$

a

$H_0 : p = 0.20 \quad H_1 : p \neq 0.20$

$P(X \leq 1) = 0.0692$

$P(X \leq 0) = 0.0115$

(0.015 nearest to 0.025)

critical value = 0

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9900 = 0.0100$

$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321$

critical value = 8 (0.0321 nearer to 0.025)

Critical region $X = 0$ and $X \geq 8$.

b $X = 8$ is in the critical region. There is enough evidence to reject H_0.
The hospital's proportion of complications differs from the national figure.

c Actual significance level is $0.0115 + 0.0321 = 0.0436$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 12

Question:

Over a number of years the mean number of hurricanes experienced in a certain area during the month of August is 4. A scientist suggests that, due to global warming, the number of hurricanes will have increased, and proposes to do a hypothesis test based on the number of hurricanes this year.

a Suggest suitable hypotheses for this test.

b Find to what level the number of hurricanes must increase for the null hypothesis to be rejected at the 5% level of significance.

c The actual number of hurricanes this year was 8. What conclusion did the scientist come to?

Solution:

Po(4)

a $H_0: \lambda = 4$ $H_1: \lambda > 4$

b $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9489 = 0.0511 > 0.05$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214 < 0.05$

Critical region is $X \geq 9$.

c 8 is not in the critical region. The scientist concluded there was not enough evidence to suggest an increase in the number of hurricanes.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 13

Question:

An estate agent usually sells properties at the rate of 10 per week.
During a recession, when money was less available, over an eight-week period he sold 55 properties.
Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the weekly rate of sales decreased.

Solution:

$H_0 : \lambda = 10 \quad H_1 : \lambda < 10$
Let Y = properties sold in 8 weeks
Under H_0 $Y \sim \text{Po}(80)$
$P(Y \leq 55) \approx P(W < 55.5)$ where $W \sim N(80, 80)$
$\approx P\left(Z < \frac{55.5 - 80}{\sqrt{80}}\right)$
$= 1 - P(Z < 2.74)$
$= 1 - 0.9970$
$= 0.0030$
$0.0030 < 0.05$ Reject H_0.
There is evidence that the rate of weekly sales has decreased.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 14

Question:

A manager thinks that 20% of his workforce are absent for at least one day each month. He chooses 100 workers at random and finds that in the last month 2 had been absent for at least one day.
Using a suitable approximation test, at the 5% level of significance, whether or not this provides evidence that the percentage of workers that are absent for at least 1 day per month is less than 20%.

Solution:

B(100, 0.2)

$H_0 : p = 0.2 \quad H_1 : p < 0.2$

$P(X \leq 2) = P(Y < 2.5)$ where $Y \sim N(20, 4^2)$

$P(Y < 2.5) = P\left(Z < \frac{2.5 - 20}{\sqrt{16}}\right)$

$= 1 - P(Z > 4.375)$

$= 0.000 < 0.05$

Reject H_0

There is evidence that the percentage of workers
who are absent for at least 1 day per month is less than 20%.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

Question:

Mai commutes to work five days a week on a train. She does two journeys a day.
Over a long period of time she finds that the train is late 20% of the time.
A new company takes over the train service Mai uses. Mai thinks that the service will be late more often. In the first week of the new service the train is late 3 times.
You may assume that the number of times the train is late in a week has a binomial distribution.
Test, at the 5% level of significance, whether or not there is evidence that there is an increase in the number of times the train is late. State your hypothesis clearly.

Solution:

$X \sim B(10, 0.20)$
$H_0 : p = 0.20 \quad H_1 : p > 0.20$
$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6778 = 0.3222$
$0.3222 > 0.05$
There is insufficient evidence to reject H_0.
There is no evidence that the trains are late more often.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

Over a long period of time it was observed that the mean number of lorries passing a hospital was 7.5 every 10 minutes.

A new by-pass was built that avoided the hospital. In a survey after the by-pass was opened, it was found that in one particular week the mean number of lorries passing the hospital was 4 every 10 minutes. It is decided that a significance test will be done to test whether or not the mean number of lorries passing the hospital has changed.

a State whether a one- or two-tailed test will be needed. Give a reason for your answer.

b Write down the name of the distribution that will be tested. Give a reason for your choice.

c Carry out the significance test at the 5% level of significance.

Solution:

a A two-tail test will be needed. We are looking to see if the number of lorries has changed.

b $X \sim \text{Po}(7.5)$ Lorries arrive independent of each other, singly and at a constant rate.

c $\text{H}_0 : \lambda = 7.5 \quad \text{H}_1 : \lambda \neq 7.5$

$\text{P}(X \leq 4) = 0.1321$

$0.1321 > 0.025$

There is insufficient evidence to reject H_0.

There is no evidence that the rate at which lorries pass the hospital has changed.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 3

Question:

A marketing company claims that Chestly cheddar cheese tastes better than Cumnauld cheddar cheese.
Five people chosen at random as they entered a supermarket were asked to say which they preferred. Four people preferred Chestly cheddar cheese.
Test, at the 5% level of significance, whether or not the company's claim is true.
State your hypothesis clearly.

Solution:

$X \sim B(5, 0.50)$
$H_0 : p = 0.50 \quad H_1 : p > 0.50$
$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.8125 = 0.1875$
$0.1875 > 0.05$
There is insufficient evidence to reject H_0. There is insufficient evidence that the company's claim is true.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 4

Question:

In 2006 and 2007 much of Greebe suffered earth tremors at a rate of 5 per month.
A survey was done in the first two months of 2008 and 13 tremors were recorded.
Stating your hypothesis clearly test, at the 10% level of significance, whether or not there is evidence to suggest the rate of earth tremors has increased.

Solution:

$X \sim \text{Po}(10)$
$\text{H}_0 : \lambda = 10 \quad \text{H}_1 : \lambda > 10$
$\text{P}(X \geq 13) = 1 - \text{P}(X \leq 12) = 1 - 0.7916 = 0.2084$
$0.2084 > 0.10$
There is insufficient evidence to reject H_0.
There is no evidence that the rate of tremors has increased.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 5

Question:

Historical information finds that nationally 30% of cars fail a brake test.

a Give a reason to support the use of a binomial distribution as a suitable model for the number of cars failing a brake test.

b Find the probability that, of 5 cars taking the test, all of them pass the brake test.

A garage decides to conduct a survey of their cars. A randomly selected sample of 10 of their cars is tested. Two of them fail the test.

c Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that cars in this garage fail less than the national average.

Solution:

a Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.

b $X \sim B(5, 0.30)$

$P(\text{all pass}) = 0.70^5 = 0.16807$

c $X \sim B(10, 0.30)$

$H_0 : p = 0.30 \quad H_1 : p < 0.30$

$P(X \leq 2) = 0.3828$

$0.3828 > 0.05$

There is insufficient evidence to reject H_0. There is no evidence that the garage fails fewer than the national average.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 6

Question:

a Explain what you understand by an hypothesis test.

During a garden fete cups of tea are thought to be sold at a rate of 2 every minute. To test this, the number of cups of tea sold during a random 30-minute interval is recorded.

b State one reason why the sale of cups of tea can be modelled by a Poisson distribution.

c Find the critical region for a two-tailed hypothesis that the number of cups of tea sold occurs at a rate of 2 every minute. The probability in each tail should be as close to 2.5% as possible.

d Write down the actual significance level of the above test.

Solution:

a A hypothesis test about a population parameter θ tests a null hypothesis H_0, which specifies a particular value for θ, against an alternative hypothesis H_1 which is that θ has increased, decreased or changed.
H_1 will indicate if the test is one-or two-tailed.

b You can count the number of cups of tea that were served in a given time interval (30 minutes). (You cannot count the number of of cups that were *not* served.

c $X \sim \text{Po}(2)$
$H_0 : \lambda = 2 \quad H_1 : \lambda \neq 2$
$P(X = 0) = 0.1353 > 0.025$
No lower critical value
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9473 = 0.0527$
$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9834 = 0.0166$
Critical value $x = 6$ since 0.0166 is closer to 0.025.
Critical region is $X \geq 6$.

d Actual level of significance = 0.0166

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Exercise D, Question 7

Question:

The probability that Jacinth manages to hit a coconut on the coconut shy at a fair is 0.4. She decides to practise at home. After practising she thinks that the practising has helped her to improve. After practising Jacinth is going to the fair and will have 20 throws.

a Find the critical region for an hypothesis test at the 5% level of significance.
After practising, Jacinth hits the coconut 11 times.

b Determine whether or not there is evidence that practising has helped Jacinth improve. State your hypothesis clearly.

Solution:

a $X \sim B(20, 0.4)$
$H_0 : p = 0.40 \quad H_1 : p > 0.40$
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.05$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.05$
Critical region is $X \geq 13$.

b $H_0 : p = 0.40 \quad H_1 : p > 0.40$
11 is not in the critical region.
There is insufficient evidence to reject H_0. There is no evidence that practice has improved Jacinth's throwing.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 8

Question:

The proportion of defective articles in a certain manufacturing process has been found from long experience to be 0.1.
A random sample of 50 articles was taken in order to monitor the production. The number of defective articles was recorded.

a Using a 5% level of significance, find the critical regions for a two-tailed test of the hypothesis that 1 in 10 articles has a defect. The probability in each tail should be as near 2.5% as possible.

b State the actual significance level of the above test.

Another sample of 20 articles was taken at a later date. Four articles were found to be defective.

c Test at the 10% significance level, whether or not there is evidence that the proportion of defective articles has increased. State your hypothesis clearly.

Solution:

a $X \sim \text{B}(50, 0.1)$
$\text{H}_0 : p = 0.10 \quad \text{H}_1 : p \neq 0.10$
$\text{P}(X \leq 1) = 0.0338$
$\text{P}(X = 0) = 0.0052$
Critical value $x = 1$ (0.0338 nearer to 0.025)
$\text{P}(X \geq 9) = 1 - \text{P}(X \leq 8) = 1 - 0.9421 = 0.0579$
$\text{P}(X \geq 10) = 1 - \text{P}(X \leq 9) = 1 - 0.9755 = 0.0245$
Critical value $x = 10$
Critical region $X \leq 1$ and $X \geq 10$.

b Actual significance level $= 0.0338 + 0.0245 = 0.0583$

c $\text{B}(20, 0.1)$
$\text{H}_0 : p = 0.1 \quad \text{H}_1 : p > 0.1$
$\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3)$
$= 1 - 0.8670$
$= 0.133$
$0.133 > 0.1$
Accept H_0. There is no evidence that the proportion of defective articles has increased.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 9

Question:

It is claimed that 50% of women use Oriels powder. In a random survey of 20 women 12 said they did not use Oriels powder.
Test at the 5% significance level, whether or not there is evidence that the proportion of women using Oriels powder is 0.5. State your hypothesis carefully.

Solution:

$X \sim \text{B}(20, 0.5)$
$\text{H}_0 : p = 0.50 \quad \text{H}_1 : p \neq 0.50$
8 used Oriels powder.
$\text{P}(X \leq 8) = 0.2517$
$0.2517 > 0.025$
There is insufficient evidence to reject H_0.
There is no evidence that the claim is wrong.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 10

Question:

A large caravan company hires caravans out for a week at a time. During winter the mean number of caravans hired is 6 per week.

a Calculate the probability that in one particular week in winter the company will hire out exactly 4 caravans.

The company decides to reduce prices in winter and do extra advertising. This results in the mean number of caravans being hired out rising to 11 per week.

b Test, at the 5% significance level, whether or not the proportion of caravans hired out has increased. State your hypothesis clearly.

Solution:

a $X \sim \text{Po}(6)$

$\text{P}(X = 4) = \text{P}(X \leq 4) - \text{P}(X \leq 3) = 0.2851 - 0.1512 = 0.1339$

b $\text{H}_0 : \lambda = 6 \quad \text{H}_1 : \lambda > 6$

$\text{P}(X \geq 11) = 1 - \text{P}(X \leq 10) = 1 - 0.9574 = 0.0426$

$0.0426 < 0.05$

There is sufficient evidence to reject H_0.

There is evidence that the rate of hiring caravans has increased.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 11

Question:

The manager of a superstore thinks that the probability of a person buying a certain make of computer is only 0.2.
To test whether this hypothesis is true the manager decides to record the make of computer bought by a random sample of 50 people who bought a computer.

a Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.2. The probability of each tail should be as close to 2.5% as possible.

b Write down the significance level of this test.

15 people buy that certain make.

c Carry out the significance test. State your hypothesis clearly.

Solution:

$X \sim \mathrm{B}(50, 0.2)$

a $\mathrm{P}(X \leq 4) = 0.0185$ (closer to 0.025)
$\mathrm{P}(X \leq 5) = 0.0480$
$c_1 = 4$
$\mathrm{P}(X \geq 16) = 1 - \mathrm{P}(X \leq 15) = 1 - 0.9692 = 0.0308$
$\mathrm{P}(X \geq 17) = 1 - \mathrm{P}(X \leq 16) = 1 - 0.9856 = 0.0144$
$c_2 = 16$ (0.0308 nearer to 0.0250)
Critical region is $X \leq 4$ and $X \geq 16$.

b Actual significance level $= 0.0185 + 0.0308 = 0.0493$

c $\mathrm{H_0} : p = 0.2 \quad \mathrm{H_1} : p \neq 0.2$
15 is not in the critical region.
There is insufficient evidence to reject $\mathrm{H_0}$.
There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 12

Question:

At one stage of a water treatment process the number of particles of foreign matter per litre present in the water has a Poisson distribution with mean 10. The water then enters a filtration bed which should extract 75% of foreign matter.
The manager of the treatment works orders a study into the effectiveness of this filtration bed. Twenty samples, each of 1 litre, are taken from the water and 64 particles of foreign matter are found.
Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the filter bed is failing to work properly.

Solution:

Po(50) since if 75% removed then $\lambda = 20 \times (0.25 \times 10)$

$H_0 : \lambda = 50 \quad H_1 : \lambda > 50$

$P(X \geq 64) \approx P(Y > 63.5)$ where $Y \sim N(50, 50)$

$\approx P\left(Z > \frac{63.5 - 50}{\sqrt{50}}\right)$

$= 1 - 0.9718$

$= 0.0282$

$0.0281 < 0.05$ Reject H_0.

There is evidence that the filter bed is failing to work properly.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 13

Question:

A shop finds that it sells jars of onion marmalade at the rate of 10 per week. During a television cookery program, onion marmalade is used in a recipe.
Over the next six weeks the shop sells 84 jars of onion marmalade. Using a suitable approximation test at the 5% significance level whether or not there is evidence that the rate of sales after the television program has increased as a result of the television program.

Solution:

Po (60)

$H_0 : \lambda = 60 \quad H_1 : \lambda > 60$

$P(X \geq 84) \approx P(Y > 83.5)$ where $Y \sim N(60, 60)$

$\approx P\left(Z > \frac{83.5 - 60}{\sqrt{60}}\right)$

$= 1 - 0.9989$

$= 0.0011$

$0.0011 < 0.05$ Reject H_0.

There is evidence that the rate of sales of onion marmalade has increased after the program.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 14

Question:

A manufacturer produces large quantities of patterned plates. It is known from previous records that 6% of the plates will be seconds because of flaws in the patterns.
To verify that the production process is not getting worse the manager takes a sample of 150 plates and finds that 15 have flaws in their patterns. Use a suitable approximation to test, at the 5% significance level, whether or not the process is getting worse.

Solution:

B(150, 0.06)
$H_0 : p = 0.06 \quad H_1 : p > 0.06$

n is large (150), p is small (0.06)
so a Poisson approximation should be used.
$\lambda = np = 150 \times 0.06 = 9$
i.e. Po (9)
Upper tail $P(X \geq 15) = 1 - P(X \leq 14)$
$= 1 - 0.9585$
$= 0.0415 < 0.05$
Reject H_0.
There is evidence that the process is getting worse.

Solutionbank S2
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Exercise D, Question 15

Question:

Jack grows apples. Over a period of time he finds that the probability of an apple being below the size required by a supermarket is 0.45.
He has recently set another orchard using a different variety of apple. A sample of 200 of this new type of apple had 60 rejected as being undersize.
Use a suitable approximation to test, at the 5% significance level, whether or not the new variety of apple is better than the old type of apple.

Solution:

B(200, 0.45)
$H_0 : p = 0.45 \quad H_1 : p < 0.45$
$P(X \le 60) = P(Y \le 60.5)$ where $Y \sim N(90, 49.5)$
$= P\left(Z < \frac{60.5 - 90}{\sqrt{49.5}}\right)$
$= P(Z < -4.19)$
$= 0.0000 < 0.05$
Reject H_0.
The new variety is better.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 1

Question:

It is estimated that 4% of people have green eyes. In a random sample of size n, the expected number of people with green eyes is 5.

a Calculate the value of n.

The expected number of people with green eyes in a second random sample is 3.

b Find the standard deviation of the number of people with green eyes in this second sample. ***E***

Solution:

a $X \sim \mathrm{B}(n, 0.04)$

X is the random variable 'number of people who have green eyes'.

X is binomial, as there is clear 'success' ($p = 0.04$) i.e. a person has green eyes or not.

Take sample size n from the question.

$$\mathrm{E}(X) = np$$
$$5 = 0.04n$$
$$n = 125$$

Use the expectation of a binomial distribution to form an equation.

Solve the equation.

b

$$\mathrm{E}(X) = 3$$
$$np = 3$$

This is a second random sample so np is different from part **a**.

$$\text{Standard deviation} = \sqrt{npq}$$
$$= \sqrt{3(1-0.04)}$$
$$= \sqrt{2.88}$$
$$= 1.70$$

The binomial distribution has a formula for the variance, npq, so square root this.

$q = 1 - p$ and $np = 3$

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 2

Question:

In a manufacturing process, 2% of the articles produced are defective. A batch of 200 articles is selected.

a Giving a justification for your choice, use a suitable approximation to estimate the probability that there are exactly 5 defective articles.

b Estimate the probability there are less than 5 defective articles. ***E***

Solution:

a $X \sim B(200, 0.02)$

X is random variable number of defective articles.

X is binomial as there is 'success' (article is defective) and failure (article is not defective).

n is large, p is small, and $np = 4$ so use Poisson approximation.

X is approximately poisson, $\lambda = 4$

$$P(X=5) = \frac{e^{-4}4^5}{5!} = 0.1563$$

Use the formula for Poisson probability. You can also use tables: $P(X \le 5) - P(X \le 4)$ which gives the same answer.

b

$$P(X<5) = P(X \le 4) = 0.6288$$

'Less than 5' does *not* include 5. X can only take discrete values and tables *include* the value you look up.

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 3

Question:

A continuous random variable X has probability density function

$$f(x) = \begin{cases} k(4x - x^3), & 0 \le x \le 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{1}{4}$.

Find

b $E(X)$,

c the mode of X,

d the median of X.

e Comment on the skewness of the distribution.

f Sketch $f(x)$. ***E***

Solution:

a

$$\int_0^2 k(4x - x^3)\,dx = 1$$

$$k\left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 1$$

Integrating a p.d.f. over an possible values of x gives 1. Here $x = 0$ to $x = 2$.

Make sure you integrate properly!

$$k(8-4) = 1$$

Substitute the limits and take care with the arithmetic here too.

$$k = \frac{1}{4}$$

b

$$\mathrm{E}(X) = \int_0^2 x \times \frac{1}{4}(4x - x^3)\,dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$$

$$= \frac{16}{15}$$

The expectation is $\int_0^2 x\mathrm{f}(x)\,dx$.

Expanding gives $\int_0^2 (x^2 - \frac{1}{4}x^4)\,dx$.

Integrate carefully and substitute the limits.

c

$$\text{At mode, } \mathrm{f}'(x) = 0$$

$$4 - 3x^2 = 0$$

$$x = \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= 1.15 \text{ (3 s.f.)}$$

The mode is the value of x at the maximum of $\mathrm{f}(x)$, i.e. the highest point of the graph.

$$\mathrm{f}(x) = \frac{1}{4}(4x - x^3)$$

$$\mathrm{f}'(x) = \frac{1}{4}(4 - 3x^2)$$

Solving gives a surd which is 1.15 (3 s.f.).

d At median m

$$\int_0^m \frac{1}{4}(4x - x^3)\,dx = \frac{1}{2}$$

$$\frac{1}{4}(2m^2 - \frac{1}{4}m^4) = \frac{1}{2}$$

$$m^4 - 8m^2 + 8 = 0$$

$$m^2 = 4 \pm 2\sqrt{2}$$

$$m = 1.08$$

$\mathrm{F}(m) = \frac{1}{2}$ so integrate $\mathrm{f}(x)$.

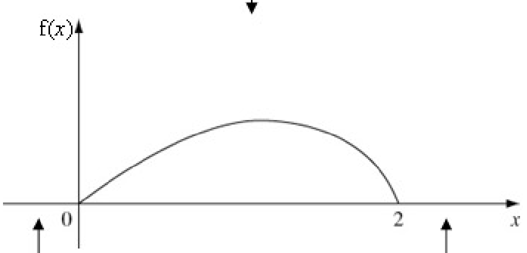
Integrate carefully and substitute m. Bottom limit is 0.

Form a quadratic in m^2 and solve using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

e mean(1.07) < median(1.08) < mode(1.15)

$\Rightarrow$ negative skew

f



Be careful as the graph is negatively skewed, but the mean, median and mode are close to each other.

Remember to add the regions $x < 0$ and $x > 2$ to your sketch.

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Review Exercise
Exercise A, Question 4

Question:

A fair coin is tossed 4 times.
Find the probability that

a an equal number of heads and tails occur,
b all the outcomes are the same,
c the first tail occurs on the third throw. *E*

Solution:

a Let X be the random variable 'the number of heads'.
$X \sim B(4, 0.5)$

Binomial distribution with number of tosses, $n = 4$ and 'success' is head, 'failure' is tail.

$$P(X=2) = C_2^4 0.5^2 \times 0.5^2$$
$$= \frac{4!}{2!2!} 0.5^2 \times 0.5^2$$
$$= 0.375$$

Use the formula for the probability of a binomial distribution.

$$\frac{4!}{2!2!} = \frac{4\times3\times2\times1}{(2\times1)\times(2\times1)} = 6$$

b $P(X=4)$ or $P(X=0)$

'Outcomes are the same'
HHHH i.e. $X = 4$
TTTT i.e. $X = 0$

$$= 2\times 0.5^4$$
$$= 0.125$$

$P(\text{HHHH}) = 0.5^4$
$P(\text{TTTT}) = 0.5^4$
'of' means add them.

c

$$P(\text{HHT}) = 0.5^3$$
$$= 0.125$$

'The first tail occurs on the third throw' means the first two outcomes must be heads so no C_r^n required.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Review Exercise

Exercise A, Question 5

Question:

Accidents on a particular stretch of motorway occur at an average rate of 1.5 per week.

a Write down a suitable model to represent the number of accidents per week on this stretch of motorway.

Find the probability that

b there will be 2 accidents in the same week,

c there is at least one accident per week for 3 consecutive weeks,

d there are more than 4 accidents in a two-week period. ***E***

Solution:

a Let X be the random variable, 'the number of accidents per week'.

$X \sim \text{Po}(1.5)$

'Rate' used in the question indicates this is a Poisson model.

b

$$P(X=2) = \frac{e^{-1.5}1.5^2}{2} = 0.2510 = 0.251 \text{ (3 s.f.)}$$

This is the formula for a Poisson probability. You can also use tables to calculate $P(X \le 2) - P(X \le 1)$.

c

$$P(X \ge 1) = 1 - P(X=0) = 1 - e^{-1.5} = 0.7769$$

'At least one' so we want 'greater than or equal to 1'.

$X = 0$ is the only unknown not required.

P (at least one accident per week for 3 weeks)

$$= 0.7769^3 = 0.4689 = 0.469 \text{ (3 s.f.)}$$

We want first week *and* second week *and* third week.

d $X \sim \text{Po}(3)$

$$P(X > 4) = 1 - P(X \le 4) = 1 - 0.8153 = 0.1847 = 0.185 \text{ (3 s.f.)}$$

'More than 4' so 4 not included in the answer.

Use tables to find $P(X \le 4)$ and subtract from 1.

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Review Exercise
Exercise A, Question 6

Question:

The random variable $X \sim B(150, 0.02)$. Use a suitable approximation to estimate $P(X > 7)$. ***E***

Solution:

$X \sim Po(3)$

n is large, p is small.
$np = 150 \times 0.02 = 3$
so use Poisson approximation.

$$\begin{aligned} P(X > 7) &= 1 - P(X \le 7) \\ &= 1 - 0.9881 \\ &= 0.0119 \text{ (3 s.f.)} \end{aligned}$$

Use tables to look up $P(X \le 7)$ and subtract from 1.

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Review Exercise
Exercise A, Question 7

Question:

A continuous random variable X has probability density function $\mathrm{f}(x)$ where,

$$\mathrm{f}(x) = \begin{cases} kx(x-2), & 2 \le x \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{4}$.

Find

b $\mathrm{E}(X)$,

c the cumulative distribution function $\mathrm{F}(x)$.

d Show that the median value of X lies between 2.70 and 2.75. ***E***

Solution:

a

$$\int_2^3 kx(x-2)\,dx = 1$$ ← $\int_2^3 f(x) = 1$

$$k\left[\frac{1}{3}x^3 - x^2\right]_2^3 = 1$$ ← $f(x) = k(x^2 - 2x)$ then integrate.

$$k\left[(9-9) - \left(\frac{8}{3} - 4\right)\right] = 1$$

$$k\left(\frac{4}{3}\right) = 1$$ ← Don't forget to substitute the bottom limit and subtract.

$$k = \frac{3}{4}$$

b

$$E(x) = \int_2^3 \frac{3}{4}x^2(x-2)\,dx$$ ← Use $E(X) = \int x f(x)\,dx$.

$$= \left[\frac{3}{16}x^4 - \frac{1}{2}x^3\right]_2^3$$ ← $xf(x) = \frac{3}{4}x^3 - \frac{3}{2}x^2$

$$= \left(\frac{3}{16}\times 3^4 - \frac{1}{2}\times 3^3\right) - \left(\frac{3}{16}\times 2^4 - \frac{1}{2}\times 2^3\right)$$ ← Substitute the upper limit, 3, and subtract the substitution of the lower limit, 2.

$$= 2\frac{11}{16} \text{ or } 2.6875$$ ← This is the *exact* answer.

c

$$F(x) = \int_2^x \frac{3}{4}(t^2 - 2t)\,dt$$ ← Use a variable upper limit with $\int f(t)\,dt$.

$$= \left[\frac{3}{4}\left(\frac{1}{3}t^3 - t^2\right)\right]_2^x$$ ← Don't forget lower limit of 2.

$$= \left(\frac{3}{4}\left(\frac{1}{3}x^3 - x^2\right) - \frac{3}{4}\left(\frac{1}{3}\times 2^3 - 2^2\right)\right)$$

$$= \frac{1}{4}(x^3 - 3x^2 + 4)$$

$$F(x) = \begin{cases} 0 & x \le 2 \\ \frac{1}{4}(x^3 - 3x^2 + 4) & 2 < x < 3 \\ 1 & x \ge 3 \end{cases}$$ ← Display your answer carefully and don't forget $F(x) = 0$ and $f(x) = 1$.

d Look at $F(x)$.

$F(2.70) = 0.453$

$F(2.75) = 0.527$ ← $F(m) = 0.5$ is in between these.

0.5 lies between $F(2.70)$ and $F(2.75)$ so m lies between 2.70 and 2.75 ← Be careful to write your answer clearly and do not get confused between 2.70 and $F(2.70)$ or 2.75 and $F(2.75)$.

Alternative method

$$\frac{1}{4}(m^3 - 3m^2 + 4) = \frac{1}{2}$$ ← Use your answer to *c*. The median, m, is where $F(m) = \frac{1}{2}$.

$$m^3 - 3m^2 + 2 = 0$$ ← This is a cubic, so it will be difficult to solve. You use the values given in the question and show that the left hand side changes sign.

$x = 2.75, x^3 - 3x^2 + 2 = 0.109315 > 0$

$x = 2.70, x^3 - 3x^2 + 2 = -0.187 < 0$

Root between 2.70 and 2.75 $\Rightarrow m$ between 2.70 and 2.75 since the cubic changes sign.

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Review Exercise
Exercise A, Question 8

Question:

The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

a exactly 2 faulty bolts,

b more than 3 faulty bolts.

These bolts are sold in bags of 20. John buys 10 bags.

c Find the probability that exactly 6 of these bags contain more than 3 faulty bolts. ***E***

Solution:

a Let X be the random variable 'the number of faulty bolts'.

$X \sim \mathrm{B}(20, 0.3)$

'Success' is 'faulty'. X is binomial with $n = 20$ bolts and probability of a faulty bolt, $p = 0.3$.

$$\mathrm{P}(X = 2) = \frac{20!}{18!2!}(0.3)^2(0.7)^{18} = 0.0278$$

Substitute into the formula for binomial probability, don't forget $^{20}C_2 = \frac{20!}{18!2!}$

You can use tables instead:
$\mathrm{P}(X \le 2) - \mathrm{P}(X \le 1) = 0.0355 - 0.0076 = 0.0279$

b

$$\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \le 3) = 1 - 0.1071 = 0.8929$$

Use tables for this as you would need to use the formula 4 times to work out $1 - (\mathrm{P}(X = 3) + \mathrm{P}(X = 2) + \mathrm{P}(X = 1) + \mathrm{P}(X = 0))$ and you are more likely to make a mistake.

c P (exactly 6 of these bags contain more than 3 faulty bolts)

More than 3 faulty bolts in a bag of 20 is the answer to **b**.

$$= \frac{10!}{4!6!}(0.8929)^6(0.1071)^4 = 0.0140$$

10 bags bought so $n = 10$.
Answer to b is p.
So we are finding $\mathrm{P}(X = 6)$ where $X \sim \mathrm{B}(10, p)$.

$$\frac{10!}{4!6!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{(4\times3\times2)\times(6\times5\times4\times3\times2\times1)}$$

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 9

Question:

a State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

The number of cars passing an observation point in a 10-minute interval is modelled by a Poisson distribution with mean 1.

b Find the probability that in a randomly chosen 60-minute period there will be

i exactly 4 cars passing the observation point,

ii at least 5 cars passing the observation point.

The number of other vehicles, (i.e. other than cars), passing the observation point in a 60-minute interval is modelled by a Poisson distribution with mean 12.

c Find the probability that exactly 1 vehicle, of **any type**, passes the observation point in a 10-minute period. ***E***

Solution:

a Events occur at a constant rate.
Events occur independently or randomly.
Events occur singly.

> There is no context stated in **a**, but Poisson requires an event to occur.

b Let X be the random variable 'the number of cars passing the point'

i $X \sim \text{Po}(6)$

> For 10 minutes, $\lambda = 1$
> For 60 minutes, $\lambda = 6$
> **a** suggests this is Poisson with $\lambda = 6$.

$$P(X = 4) = \frac{e^{-6}6^4}{4!} = 0.1339 = 0.134 \text{ (3 s.f.)}$$

> This is solved using the formula, but you can use tables and find $P(X \le 4) - P(X \le 3) = 0.2851 - 0.1512$.

ii

$$P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.2851 = 0.7149 = 0.715 \text{ (3 s.f.)}$$

> At least 5 means include 5 in your probability.

> Use tables here as otherwise the formula needs to be used 5 times.

c

$$\lambda = 1 + 2 = 3$$
$$P(X = 1) = 3e^{-3} = 0.149$$

> For car, $\lambda = 1$
> For others, $\lambda = 2$ in 10 minutes

> $X = 1$ is '1 vehicle of any type'.

Alternative method

$$P(1 \text{ car and } 0 \text{ other}) + P(0 \text{ car and } 1 \text{ other})$$
$$= 1e^{-1} \times e^{-2} + e^{-1} \times 2e^{-2}$$

> For 'other'
> 60-minute interval $\lambda = 12$
> 10-minute interval $\lambda = 2$
> For car, $\lambda = 1$

$$= 0.3679 \times 0.1353 + 0.3679 \times 0.2707$$
$$= 0.149$$

> 'and' means 'multiply'

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 10

Question:

The continuous random variable Y has cumulative distribution function $\mathrm{F}(y)$ given by

$$\mathrm{F}(y)=\begin{cases}0, & y<1,\\ k(y^4+y^2-2), & 1\le y\le 2,\\ 1, & y>2.\end{cases}$$

a Show that $k=\frac{1}{18}$.

b Find $\mathrm{P}(Y>1.5)$.

c Specify fully the probability density function $\mathrm{f}(y)$. ***E***

Solution:

a

$$\begin{aligned}\mathrm{F}(2) &= 1\\ k(2^4+2^2-2) &= 1\\ 18k &= 1\\ k &= \frac{1}{18}\end{aligned}$$

F(y) is the cumulative distribution function, so F(2) is found and equated to 1, the total probability.

b

$$\begin{aligned}\mathrm{P}(Y>1.5) &= 1-\mathrm{P}(Y\le 1.5)\\ &= 1-\mathrm{F}(1.5)\\ &= 1-\frac{1}{18}(1.5^4+1.5^2-2)\\ &= 0.705\left[\text{or } \frac{203}{288}\right]\end{aligned}$$

c

$$\begin{aligned}\mathrm{f}(y) &= \frac{\mathrm{d\,F}(y)}{\mathrm{d}y}\\ &= \frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{1}{18}(y^4+y^2-2)\right]\\ &= \frac{1}{18}(4y^3+2y)\\ &= \frac{1}{9}(2y^3+y), 1\le y\le 2\end{aligned}$$

Differentiate the c.d.f. to get the p.d.f.

$$\mathrm{f}(y)=\begin{cases}0, \text{ otherwise}\\ \frac{1}{9}(2y^3+y), & 1\le y\le 2\end{cases}$$

Set out f(y) clearly and don't forget $\mathrm{f}(y)=0$.

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Review Exercise
Exercise A, Question 11

Question:

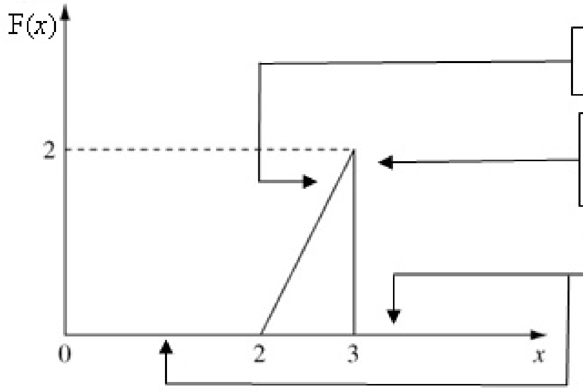
The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} 2(x-2), & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch f(x) for all values of x.
b Write down the mode of X.
Find
c E(X),
d the median of X.
e Comment on the skewness of this distribution. Give a reason for your answer. ***E***

Solution:

a



$f(x) = 2(x-2)$ is a straight line.

Make it clear that the maximum height is $f(x) = 2$.

Don't forget $x > 3$ and $x < 2$.

b Mode of X is 3.

This is the value of x where $f(x)$ is at its greatest value.

c

$$E(x) = \int_2^3 2x(x-2)\,dx$$

$$= \left[\frac{2x^3}{3} - 2x^2\right]_2^3$$

$$= 2\tfrac{2}{3}$$

Use $\int_2^3 x f(x)\,dx$.

Integrate after expanding to $2x^2 - 4x$.

d

$$\int_2^m 2(x-2)\,dx = 0.5$$

$$(x^2 - 4x)_2^m = 0.5$$

$$m^2 - 4m + 4 = 0.5$$

$$2m^2 - 8m + 7 = 0$$

F(m) = 0.5 for median.

$$m = \frac{8 \pm \sqrt{64-56}}{4}$$

$$m = \frac{4 \pm \sqrt{2}}{2}$$

$$m = 2.71$$

Solve using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and cancel by 2.

Ignore $m = 1.29$ as outs the $2 \le x \le 3$.

e Negative skew

mean(2.6) < median(2.71) < mode(3)

$E(x)$ is the mean, $2\tfrac{2}{3}$.

Solutionbank S2

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Review Exercise
Exercise A, Question 12

Question:

An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty.
Faulty components are detected at a rate of 1.5 per hour.

a Suggest a suitable model for the number of faulty components detected per hour.

b Describe, in the context of this question, two assumptions you have made in part **a** for this model to be suitable.

c Find the probability of 2 faulty components being detected in a 1-hour period.

d Find the probability of at least one faulty component being detected in a 3–hour period. ***E***

Solution:

a Let x be the random variable 'number of faulty components detected'
$X \sim \text{Po}(1.5)$

b *Faulty* components occur at a constant rate.
Faulty components occur independently and randomly.
Faulty components occur singly.

Make sure you write about the context of *faulty* components.

c
$$P(X=2) = \frac{e^{-1.5}(1.5)^2}{2!} = 0.251$$

Use the formula for the probability of a Poisson distribution with $\lambda = 1.5$. You could also use tables and $P(X \le 2) - P(X \le 1)$.

d $X \sim \text{Po}(4.5)$

Three-hour period, so $\lambda = 3 \times 1.5 = 4.5$

$$\begin{aligned} P(X \ge 1) &= 1 - P(X=0) \\ &= 1 - e^{-4.5} \\ &= 1 - 0.0111 \\ &= 0.9889 \\ &= 0.989 \text{ (3 s.f.)} \end{aligned}$$

'At least 1' so 1 is included in the probability.

Use formula for Poisson.

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Review Exercise
Exercise A, Question 13

Question:

a Write down the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01.

b Find the probability that 2 consecutive calls will be connected to the wrong agent.

c Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.

The call centre receives 1000 calls each day.

d Find the mean and variance of the number of wrongly connected calls.

e Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent. ***E***

Solution:

a If $X \sim \mathrm{B}(n, p)$ and
n is large
p is small
then X can be approximated by $\mathrm{Po}(np)$.

b

$$\mathrm{P}(2 \text{ consecutive calls}) = 0.01^2 = 0.0001$$

c $X \sim \mathrm{B}(5, 0.01)$

'Success' is 'connected to wrong agent' number of trials, $n = 5$.

$$\mathrm{P}(X > 1) = 1 - \mathrm{P}(X = 1) - \mathrm{P}(X = 0) = 1 - 5(0.01)(0.99)^4 - (0.99)^5 = 0.00098$$

'More than 1' means 1 is not included in the probability.

d $X \sim \mathrm{B}(1000, 0.01)$

$n = 1000$ calls per day $p = 0.01$ probability of a wrongly connected call

mean $= np = 10$
variance $= np(1-p) = 9.9$

Use formulae for mean and variance of binomial distribution.

e $X \sim \mathrm{Po}(10)$

$np = 10$ from **d**.

$$\mathrm{P}(X > 6) = 1 - \mathrm{P}(X \le 6) = 1 - 0.1301 = 0.8699 = 0.870 \text{ (3 s.f.)}$$

'More than 6' means 6 is not included.

Look up 6 in Poisson tables with $\lambda = 10$ and subtract from 1.

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Review Exercise
Exercise A, Question 14

Question:

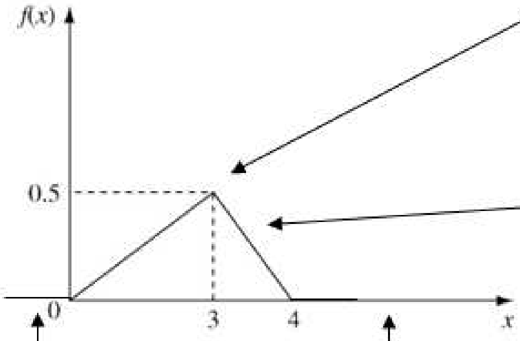
The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x, & 0 < x < 3, \\ 2 - \frac{1}{2}x, & 3 \le x < 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the probability density function of X.
b Find the mode of X.
c Specify fully the cumulative distribution function of X.
d Using your answer to part **c**, find the median of X. ***E***

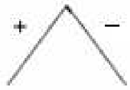
Solution:

a



Make sure you draw each part as a straight line.

$\frac{1}{6}x$ has positive gradient



$2-\frac{1}{2}x$ has negative gradient.

Don't forget $x \le 0$
$x \ge 4$

b Mode is $x = 3$.

Mode is the value of x at the highest point of $f(x)$ on the sketch.

c

$$F(x) = \int_0^x \frac{1}{6}t\ dt = \frac{1}{12}x^2, 0 < x \le 3$$

$\int f(t)\,dt$ up to $x = 3$

$$F(x) = \int_3^x \left(2-\frac{1}{2}t\right) dt + \int_0^3 \frac{1}{6}t\ dt$$
$$= \left(2x-\frac{1}{4}x^2\right) - \left(2\times 3-\frac{1}{4}\times 3^2\right) + \frac{1}{12}\times 3^2$$
$$= 2x-\frac{1}{4}x^2-3, \quad 3 < x < 4$$

Don't forget the area between $x = 0$ and $x = 3$.

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{12}x & 0 < x \le 3 \\ 2x-\frac{1}{4}x^2-3 & 3 < x < 4 \\ 1 & x \ge 4 \end{cases}$$

Don't forget the ends of the c.d.f.

d

$$F(m) = 0.5$$

Median is 'half-way'.

$$\frac{1}{12}m^2 = 0.5$$

$F(3) = \frac{3}{4}$, so m is between 0 and 3.

$$m = \sqrt{6} = 2.45$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Review Exercise

Exercise A, Question 15

Question:

The random variable J has a Poisson distribution with mean 4.

a Find $P(J \geq 10)$

The random variable K has a binomial distribution with parameters $n = 25$, $p = 0.27$.

b Find $P(K \leq 1)$

Solution:

a

$$\begin{aligned} P(J \geq 10) &= 1 - P(J \leq 9) \\ &= 1 - 0.9919 \\ &= 0.0081 \end{aligned}$$

$J \sim Po(4)$

Value from tables $n = 10, \lambda = 4$

b

$$\begin{aligned} P(K \leq 1) &= P(K = 0) + P(K = 1) \\ &= (0.73)^{25} + 25(0.73)^{24}(0.27) \\ &= 0.00392 \end{aligned}$$

$K \sim B(25, 0.27)$

Use formula for binomial probability.

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 16

Question:

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 2x^2 - x^3, & 0 \le x \le 1, \\ 1, & x > 1. \end{cases}$$

a Find $P(X > 0.3)$.

b Verify that the median value of X lies between $x = 0.59$ and $x = 0.60$.

c Find the probability density function $f(x)$.

d Evaluate $E(X)$.

e Find the mode of X.

f Comment on the skewness of X.
Justify your answer.

Solution:

a

$$P(X > 0.3) = 1 - F(0.3)$$
$$= 1 - (2 \times 0.3^2 - 0.3^3)$$
$$= 0.847$$

Remember to 'one minus' as we want $X > 0.3$.

b

$$F(0.59) = 0.4908 < 0.5$$
$$F(0.60) = 0.5040 > 0.5$$

0.5 lies between F(0.59) and F (0.60)
so median lies between 0.59 and 0.60

'Verify' so write your answer clearly.

c

$$f(x) = \frac{dF(x)}{dx}$$
$$= \frac{d}{dx}(2x^2 - x^3)$$
$$f(x) = 4x - 3x^2, 0 \le x \le 1$$
$$f(x) = 0, \text{otherwise}$$

Differentiate c.d.f. to find p.d.f.

Remember $x < 0$ and $x > 1$.

$$f(x) = \begin{cases} 4x - 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

d

$$E(X) = \int_0^1 x f(x)\, dx$$
$$= \int_0^1 (4x^2 - 3x^3)\, dx$$
$$= \left[4\frac{x^3}{3} - 3\frac{x^4}{4}\right]_0^1$$
$$= \frac{7}{12} \text{ or } 0.58\dot{3}$$

Bottom limit substitutes to give 0.

$$\frac{4}{3} - \frac{3}{4} = \frac{7}{12}$$

e $\frac{df(x)}{dx} = -6x + 4$

For mode,
$$-6x + 4 = 0$$
$$x = \frac{2}{3} \text{ or } 0.\dot{6}$$

Mode occurs at maximum value of $f(x)$ where $\frac{df(x)}{dx} = 0$.

f mean$(0.58\dot{3})$ < median$(0.59 - 0.6)$ < mode$(0.\dot{6})$
so negative skew

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 1

Question:

The random variable X is uniformly distributed over the interval $[-1, 5]$.

a Sketch the probability density function, $f(x)$, of X.

Find

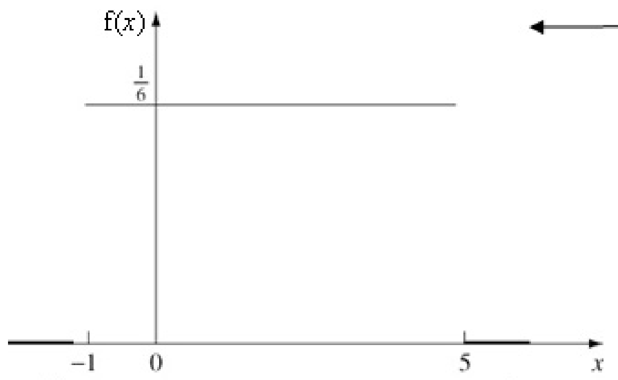
b $E(X)$,

c $Var(X)$,

d $P(-0.3 < X < 3.3)$. ***E***

Solution:

a



$f(x) = \frac{1}{5+1} = \frac{1}{6}$
for $-1 \le x \le 5$

Label the key values $x = -1, x = 5$ and $f(x) = \frac{1}{6}$.

Don't forget $x > 5$ and $x < -1$.

b

$$E(X) = -1 + 3 = 2 \text{ or}$$

$$E(X) = \frac{-1+5}{2} = 2$$

Use symmetry rather than integration.

c

$$Var(X) = \frac{1}{12}(5+1)^2 = 3$$

Use the formula for a uniform distribution rather than integration.

d

$$P(-0.3 < X < 3.3) = 3.6 \times \frac{1}{6} = 0.6$$

Use *area under graph* from $x = -0.3$ to $x = 3.3$ rather than definite integration. It is easier!

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 2

Question:

A bag contains a large number of coins. Half of them are 1p coins, one third are 2p coins and the remainder are 5p coins.

a Find the mean and variance of the value of the coins.

A random sample of 2 coins is chosen from the bag.

b List all the possible samples that can be drawn.

c Find the sampling distribution of the mean value of these samples. *E*

Solution:

a

X	1	2	5
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Draw a probability distribution table to help with calculations.

$\text{Mean} = 1\times\frac{1}{2}+2\times\frac{1}{3}+5\times\frac{1}{6}=2$

Use $\Sigma x\text{P}(X=x)$

Mean value is 2p or £0.02

$$\text{Variance} = 1^2\times\frac{1}{2}+2^2\times\frac{1}{3}+5^2\times\frac{1}{6}-2^2$$
$$= 2$$

Use $\Sigma x^2\text{P}(X=x)-\mu^2$

Variance has units of $(\text{pence})^2$.

b (1, 1)
(1, 2) and (2, 1)
(1, 5) and (5, 1)
(2, 2)
(2, 5) and (5, 2)
(5, 5)

2 coins drawn so easy to list as (first coin, second coin)

Write down the list carefully and don't miss any possible samples.

c

$\bar{x}$	1	1.5	2	3	3.5	5
$P(X=x)$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$2\times\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}$	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$	$\frac{1}{6}$	$2\times\frac{1}{3}\times\frac{1}{6}=\frac{1}{9}$	$\frac{1}{36}$

Distribution means set out in a table.

Possible means are

$$\frac{1+1}{2} = 1,\ \frac{1+2}{2}=1.5$$
$$\frac{1+5}{2} = 3,\ \frac{2+2}{2}=2$$
$$\frac{2+5}{2} = 3.5,\ \frac{5+5}{2}=5$$

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise

Exercise A, Question 3

Question:

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.
He chooses 20 pupils at random and finds 9 of them read Deano.

a **i** Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read Deano is different from 20%. State your hypotheses clearly.

ii State all the possible numbers of pupils that read Deano from a sample of size 20 that will make the test in part **a i** significant at the 5% level.

The teacher takes another 4 random samples of size 20 and they contain 1, 3, 1 and 4 pupils that read Deano.

b By combining all 5 samples and using a suitable approximation test, at the 5% level of significance, whether or not this provides evidence that the percentage of pupils in the school that read Deano is not 20%.

c Comment on your results for the tests in part **a** and part **b**. ***E***

Solution:

a **i**

$H_0 : p = 0.2$
$H_1 : p \neq 0.2$

Two-tailed test as 'is different from'

$P(X \geq 9) = 1 - P(x \leq 8)$
$= 1 - 0.9900$
$= 0.0100 < 0.025$

$X \sim B(20, 0.2)$ so use tables to look up $P(X \leq 8)$.

Compare with 0.025 at 5%, two-tailed test.

Reject H_0.
Evidence that the percentage of pupils that read Deano is not 20%, it is more than 20%.

State conclusion in context from question.

ii From tables

$P(X = 0) = 0.0115 < 0.025$
$P(X \leq 1) = 0.0692 > 0.025$
$P(X \geq 8) = 1 - 0.9679 = 0.032 > 0.025$
$P(X \geq 9) = 0.0100 < 0.025$

All possible values are 0 or [9, 20] or 0 and 9 or more.

Compare with 0.025 so include 0.

8 not included as > 0.025.

Upper limit is sample size of 20.

b

$H_0 : p = 0.2$
$H_1 : p \neq 0.2$

Two-tailed test as question says 'is not 20%'.

$W \sim B(100, 0.2)$
$W \sim N(20, 16)$
$\text{Total} = 9 + 1 + 3 + 1 + 4 = 18$

Use normal approximation to binomial, N(*np*, *npq*).

$$P(X \leq 18) = P\left(Z \leq \frac{18.5 - 20}{4}\right)$$
$$= P(Z \leq -0.375)$$
$$= 0.352$$

Include 18 so use continuity correction +0.5 then standardise.

Use tables: you do not need to interpolate.

$0.352 > 0.025$ so insufficient evidence to reject H_0.
Combined numbers of Deano readers suggests there is no reason to doubt 20% of pupils read Deano.

Write conclusion in context.

c

In part **a** we *rejected* H_0
In part **b** we had *insufficient evidence* to reject H_0.
The results are *different*.
Either sample size matters and
larger samples give more reliable results
or not all pupils are drawn from the same population.

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2 Review Exercise
Exercise A, Question 4

Question:

The continuous random variable X is uniformly distributed over the interval [2, 6].

a Write down the probability density function f(x).

Find

b E(X),

c Var(X),

d the cumulative distribution function of X, for all x,

e $P(2.3 < X < 3.4)$. *E*

Solution:

a $f(x) = \begin{cases} \frac{1}{4}, & 2 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$

$\frac{1}{6-2} = \frac{1}{4}$

b $E(x) = 4$

By symmetry half way between 2 and 6.

c

$$\text{Var}(X) = \frac{(6-2)^2}{12} = \frac{4}{3}$$

Using variance formula for a uniform distribution.

d

$$F(x) = \int_2^x \frac{1}{4}\,dt = \left[\frac{1}{4}t\right]_2^x = \frac{1}{4}(x-2)$$

Remember $F(x) = \int^x f(x)\,dx$ but use t with a variable upper limit of x.

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{4}(x-2), & 2 \le x \le 6 \\ 1, & x > 6 \end{cases}$$

Remember the ends.

e

$$P(2.3 < X < 3.4) = \frac{1}{4}(3.4-2.3) = 0.275$$

Area of rectangle under f(x), height $\frac{1}{4}$.

Alternative method

$$P(2.3 < X < 3.4) = F(3.4) - F(2.3) = \frac{1}{4}(3.4-2) - \frac{1}{4}(2.3-2) = 0.275$$

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2 Review Exercise

Exercise A, Question 5

Question:

The random variable X is the number of misprints per page in the first draft of a novel.

a State two conditions under which a Poisson distribution is a suitable model for X.

The number of misprints per page has a Poisson distribution with mean 2.5. Find the probability that

b a randomly chosen page has no misprints,

c the total number of misprints on 2 randomly chosen pages is more than 7.

The first chapter contains 20 pages.

d Using a suitable approximation find, to 2 decimal places, the probability that the chapter will contain fewer than 40 misprints. ***E***

Solution:

a Misprints occur randomly and independently.
Misprints occur singly in space.
Misprints occur at a constant rate.

Use the context of misprints.

b

$$P(X=0) = e^{-2.5}$$
$$= 0.0821$$

Question gives $Po(2.5)$.

c For 2 pages, $Y \sim Po(5)$

$\lambda = 2 \times 2.5 = 5$

$$P(Y > 7) = 1 - P(Y \leq 7)$$
$$= 1 - 0.8666 = 0.1334$$
$$= 0.133 (3 \text{ s.f.})$$

Question says 'more than 7' so 7 not included.

Use tables of $Po(5)$.

d For 20 pages, $Y \sim Po(50)$
$Y \sim N(50, 50)$ approx

$\lambda = 20 \times 2.5 = 50$

Satisfies normal approximation to Poisson.

$$P(Y < 40) = P(Y \leq 39.5)$$

'Less than 40' so 40 not included so include continuity correction by $40 - 0.5 = 39.5$.

$$= P\left(Z \leq \frac{39.5 - 50}{\sqrt{50}}\right)$$
$$= P(Z \leq -1.4849)$$
$$= 1 - 0.93 = 0.07$$

Standardise then use normal distribution tables to look up 2 values.

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2 Review Exercise
Exercise A, Question 6

Question:

Explain what you understand by

a a sampling unit,

b a sampling frame,

c a sampling distribution. ***E***

Solution:

a *Element* of the *population*.

b A *list of all* the sampling units. ← A sampling frame is a list, register or database of sampling units.

c *All possible samples* are chosen from a population, the *values* of a statistic and the associated *probabilities* is a sampling distribution. ← Usually displayed in a table.

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2 Review Exercise
Exercise A, Question 7

Question:

A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.
A random sample of 10 patients with depression is taken from a doctor's records.

a Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

Given that the claim is correct,

b find the probability that the treatment will be successful for exactly 6 patients.

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

c Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.

d From a sample of size 20, find the greatest number of patients who need to recover for the test in part **c** to be significant at the 1% level. ***E***

Solution:

a $X \sim B(10, 0.75)$

where X is the random variable 'number of patients who recover when treated'

> p is the probability that a patient recovers in the random sample we are told it is claimed $p = 0.75$.

b

$$P(X = 6) = P(X \le 6) - P(X \le 5)$$
$$= 0.9219 - 0.7759$$
$$= 0.146$$

> 'Claim is correct' means take $p = 0.75$.

Alternative method

$$P(X = 6) = \frac{10!}{6!4!} \times 0.75^6 \times 0.25^4$$
$$= 0.146$$

> Use the binominal formula.

c

$H_0 : p = 0.75$

$H_1 : p < 0.75$

$X \sim B(20, 0.75)$

> One-tailed test as 'lower' in question.

$$P(X \le 13) = 1 - 0.7858$$
$$= 0.2142 > 0.05$$

> Looking at 'left hand tail'.

> One-tailed test so compare with 5%.

Insufficient evidence to reject H_0.

Doctor's belief is not supported.

> Remember the context of 'doctor's belief'.

d

$$P(X \le 9) = 1 - 0.9961 = 0.0039 < 0.01$$
$$P(X \le 10) = 1 - 0.9861 = 0.0139 > 0.01$$

> Look at values in B(20, 0.75) table.
> Compare with 0.01, i.e. 1% significance.

So greatest number of patients is 9.

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2 Review Exercise
Exercise A, Question 8

Question:

a Explain what you understand by a census.
Each cooker produced at GT Engineering is stamped with a unique serial number. GT Engineering produces cookers in batches of 2000. Before selling them, they test a random sample of 5 to see what electric current overload they will take before breaking down.
b Give one reason, other than to save time and cost, why a sample is taken rather than a census.
c Suggest a suitable sampling frame from which to obtain this sample.
d Identify the sampling units. ***E***

Solution:

a A census is when *every member* of a *population* is investigated.
b This is destructive testing, so there would be no cookers left to sell if a census were taken.

'Cheap' or 'quick' is not enough for this part.

c A *list* of the serial numbers of the cookers.

Register/database is also OK for list.

d A cooker.

Serial number of a cooker is OK too.

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2 Review Exercise
Exercise A, Question 9

Question:

Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.
Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly. ***E***

Solution:

$H_0 : p = 0.3 \quad H_1 : p > 0.3$ ← One-tailed test as question says 'increased'.

X is the random variable
'number of tomatoes greater than 4 cm'
$X \sim B(40, 0.3)$ ← Samples size n, $n = 40, p = 0.3$

$$
\begin{aligned}
P(X \geq 18) &= 1 - P(X \leq 17) \\
&= 1 - 0.9680 \\
&= 0.0320 < 0.05
\end{aligned}
$$
← One-tailed test at 5% significance level.

Reject H_0
Dhiriti's claim is supported by sample. ← Remember context, you could write out the claim from the question too.

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2 Review Exercise
Exercise A, Question 10

Question:

The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

a Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

i a Poisson approximation,

ii a Normal approximation.

b Write down which of the approximations used in part **a** is a more accurate estimate of the probability.
You must give a reason for your answer. ***E***

Solution:

a Let X be the random variable 'the number of sunflower plants more than 1.5 m high'

i $X \sim \text{Po}(10)$ ← $\lambda = 40 \times 0.25 = 10$

$$\begin{aligned} P(8 \le X \le 13) &= P(X \le 13) - P(X \le 7) \\ &= 0.8645 - 0.2202 \\ &= 0.6443 \\ &= 0.644 \text{ (3 s.f.)} \end{aligned}$$

Inclusive, so subtract $P(X \le 7)$.

Use tables for these values.

ii $X \sim N(10, 7.5)$ ← $np = 40 \times 0.25 = 10$, $npq = 10 \times 0.75 = 7.5$

$$P(7.5 \le X \le 13.5) = P\left(\frac{7.5-10}{\sqrt{7.5}} \le Z \le \frac{13.5-10}{\sqrt{7.5}}\right)$$

Inclusive so $8 - 0.5 = 7.5$, $13 + 0.5 = 13.5$ for continuity correction.

Don't forget to square root 7.5.

$$\begin{aligned} &= P(-0.913 \le Z \le 1.278) \\ &= 0.8997 - (1 - 0.8186) \\ &= 0.7183 \\ &= 0.718 \text{ (3 s.f.)} \end{aligned}$$

Use normal tables.

-0.913 so use '$1-$'

b Normal approximation as n large, p close to $\frac{1}{2}$ and $np = 10 > 5$.
(Exact binomial is 0.7148 $(0.8968 - 0.1820)$.)
The Poisson approximation shouldn't be used because p isn't small, it is bigger than 0.1.

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2 Review Exercise
Exercise A, Question 11

Question:

a Explain what you understand by
  i an hypothesis test,
  ii a critical region.

During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20-minute interval, is recorded.

b Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1-minute interval. The probability in each tail should be as close to 2.5% as possible.

c Write down the actual significance level of the above test.

In the school holidays, 1 call occurs in a 10-minute interval.

d Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time. ***E***

Solution:

a

i A hypothesis test is where the value of a population parameter (whose assumed value is given in H_0) is tested against what value it takes if H_0 is rejected (this could be an increase, a decrease or a change).

ii A *range of values of a test stanstic* that would lead to the *rejection of the null hypothesis*.

b Let X be the random variable 'the number of incoming calls'

$X \sim \text{Po}(9)$

From table

$\lambda = 20 \times 0.45 = 9$

$$\text{P}(X \geq 16) = 1 - \text{P}(X \leq 15) = 1 - 0.9780 = 0.0220 \text{ which is closest to } 0.025$$

$\text{P}(X \geq 15) = 1 - (X \leq 14) = 0.0415 > 2.5\%$

$\text{P}(X \leq 3) = 0.212$ which is closest to 0.025

$\text{P}(X \leq 4) = 0.0550 > 2.5\%$

Critical region $X \leq 3$ or $X \geq 16$

c Actual significance level
$0.0220 + 0.0212 = 0.0432$ or 4.32%

d $H_0 : \lambda = 0.45 \quad H_1 : \lambda < 0.45$

$\lambda = 4.5$

Testing 0.45 per 1-minute interval, but using $10 \times 0.45 = 4.5$ as the parameter λ to look up in the table.

$X \sim \text{Po}(4.5)$

$\text{P}(X \leq 1) = 0.0611 > 0.05$

Insufficient evidence to reject H_0.

The rate of incoming calls is *less* during the school holidays is *not* supported.

'is less' in question so one-tailed H_1 and conclusion clear.

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 12

Question:

A string AB of length 5 cm is cut, in a random place C, into two pieces. The random variable X is the length of AC.

a Write down the name of the probability distribution of X and sketch the graph of its probability density function.

b Find the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

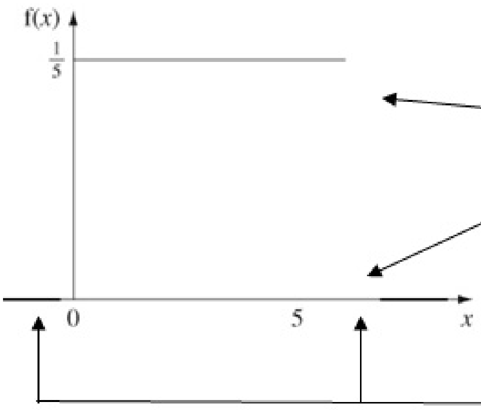
c Find $\mathrm{P}(X > 3)$.

d Write down the probability that AC is 3 cm long **E**

Solution:

a Continuous uniform distribution

Alternatively, a rectangular distribution.



All your lines should be parallel to the x-axis.

Don't forget $x < 0$ and $x > 5$.

b

$$\mathrm{E}(X) = 2.5$$

By symmetry of sketch.

$$\mathrm{Var}(X) = \frac{(5-0)^2}{12} = \frac{25}{12}$$

Use the formula rather than integration.

c

$$\mathrm{P}(X > 3) = (5-3) \times \frac{1}{5} = \frac{2}{5}$$

Area of rectangle.

d $\mathrm{P}(X = 3) = 0$

It is a continuous distribution so an exact value has probability 0.

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2 Review Exercise
Exercise A, Question 13

Question:

Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and it contains 7 bacteria. Stating your hypotheses clearly, test his claim at the 5% level of significance. ***E***

Solution:

$H_0: \lambda = 5$

$H_1: \lambda > 5$

'5 per litre of water'

'Polluting the river' so bacteria being added, so one-tailed test.

$X \sim \text{Po}(2.5)$

$$\begin{aligned} P(X \geq 7) &= 1 - P(X \leq 6) \\ &= 1 - 0.9858 \\ &= 0.0142 < 0.05 \end{aligned}$$

Use $\lambda = 5 \times 0.5 = 2.5$ table.

Reject H_0

There is significant evidence that near the factory the river is polluted with bacteria at the 5% level.

You can also say 'scientist's claim is justified'.

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2 Review Exercise
Exercise A, Question 14

Question:

A bag contains a large number of coins
75% are 10p coins,
25% are 5p coins.
A random sample of 3 coins is drawn from the bag.
Find the sampling distribution for the median of the values of the 3 selected coins. ***E***

Solution:

Possible samples of 3 coins are
(5, 5, 5)
(5, 5, 10) in any order (3 cases) ← (5, 10, 5) or (10, 5, 5)

(10, 10, 5) in any order (3 cases) ← (10, 5, 10) or (5, 10, 10)
(10, 10, 10)

8 cases altogether with median of 5 or 10.
Median 5 ← (5, 5, 5) and (5, 5, 10) have median of 5.

$$P(M=5) = \left(\frac{1}{4}\right)^3 + 3\times\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$$
$$= \frac{10}{64} \text{ or } \frac{5}{32}$$

Remember the 3 cases.

Median 10

$$P(M=10) = \left(\frac{3}{4}\right)^3 + 3\times\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$$
$$= \frac{54}{64} \text{ or } \frac{27}{32}$$

(5, 10, 10) and (10, 10, 10) have median of 10.

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2 Review Exercise
Exercise A, Question 15

Question:

Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week with the new driver, the taxi is late 3 times.
You may assume that the number of times a taxi is late in a week has a binomial distribution.
Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly. ***E***

Solution:

$H_0 : p = 0.2$ ← $p = \frac{1}{5} = 0.2$

$H_1 : p > 0.2$ ← 'late more often' so one-tail test

$X \sim B(5, 0.2)$ ← 5 days, $P(\text{late}) = 0.2$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.9421$ ← Use tables.

$= 0.0579 > 0.05$

Insufficient evidence to reject H_0.
No evidence of an increase in the number of times the taxi driver is late. ← Remember context of 'taxi driver is late' or you could say 'Linda's claim is not justified'.

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2 Review Exercise
Exercise A, Question 16

Question:

a **i** Write down two conditions for $X \sim B(n, p)$ to be approximated by a normal distribution $Y \sim N(\mu, \sigma)$.

ii Write down the mean and variance of this normal approximation in terms of n and p.

A factory manufactures 2000 DVDs every day. It is known that 3% of DVDs are faulty.

b Using a normal approximation, estimate the probability that at least 40 faulty DVDs are produced in one day.

The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs £0.70 to manufacture a DVD and the factory sells non-faulty DVDs for £11.

c Find the expected profit made by the factory per day. ***E***

Solution:

a **i** If $X \sim B(n, p)$
n is large
p is close to 0.5
and $np > 5$
then X can be approximated by
$N(np, np(1-p))$

ii
$$\text{mean} = np$$
$$\text{variance} = npq = np(1-p)$$

'In terms of n and p'

b $X \sim N(60, 58.2)$

$np = 2000 \times 0.03 = 60$
$npq = 2000 \times 0.03 \times 0.97 = 58.2$

$$\begin{aligned} P(X \geq 40) &= P(X > 39.5) \\ &= 1 - P\left(Z \leq \frac{39.5 - 60}{\sqrt{58.2}}\right) \\ &= 1 - P(Z < -2.687\ldots) \\ &= 0.9964 \\ &= 0.996 \text{ (3 s.f.)} \end{aligned}$$

40 included so continuity correction $40 - 0.5 = 39.5$

Don't forget the square root of the variance.

c $E(X) = 60$

$np = 60$ above.

Expected profit
$$= (2000 - 60) \times 11 - 2000 \times 0.70$$
$$= £19\,940$$

Profit from sales.

Manufacturing costs

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2 Review Exercise
Exercise A, Question 17

Question:

a Define a statistic.

A random sample $X_1, X_2, \ldots, X_n$ is taken from a population with unknown mean μ.

b For each of the following state whether or not it is a statistic.

i $\frac{X_1 + X_4}{2}$,

ii $\frac{\Sigma X^2}{n} - \mu^2$. ***E***

Solution:

a A random variable that is a function of known observations from a population.
or A statistic is a numerical property of a sample.

b **i** Yes, it is a statistic.

X_1 and X_2 are known.

ii No, it is not a statistic.

μ is unknown.

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2 Review Exercise
Exercise A, Question 18

Question:

For a particular type of plant 45% have white flowers and the remainder have coloured flowers. Gardenmania sells plants in batches of 12. A batch is selected at random.
Calculate the probability this batch contains

a exactly 5 plants with white flowers,

b more plants with white flowers than coloured ones.

Gardenmania takes a random sample of 10 batches of plants.

c Find the probability that exactly 3 of these batches contain more plants with white flowers than coloured ones.

Due to an increasing demand for these plants by large companies, Gardenmania decides to sell them in batches of 50.

d Use a suitable approximation to calculate the probability that a batch of 50 plants contains more than 25 plants with white flowers. ***E***

Solution:

a Let W be the random variable 'the number of white plants'

$W \sim B(12, 0.45)$

'batches of 12': $n = 12$
'45% have white flowers'

$P(W = 5) = C_5^{12} 0.45^5 0.55^7$

$= 0.2225$

You can also use tables:
$P(W \le 5) - P(W \le 4)$

b

$P(W \ge 7) = 1 - P(W \le 6)$

$= 1 - 0.7393$

$= 0.2607$

Batches of 12, so
7 white, 5 coloured
8 white, 4 coloured, etc.

c

$P(\text{exactly } 3) = C_3^{10} (0.2607)^3 (1 - 0.2607)^7$

$= 0.2567$

Use your answer to **b**:
$p = 0.2607, n = 10$

d

mean $= np = 22.5$

variance $= npq = 12.375$

$m = 50$
$p = 0.45$
$q = 0.55$

$W \sim N(22.5, 12.375)$

$P(W > 25) = P\left(Z > \frac{25.5 - 22.5}{\sqrt{12.375}}\right)$

25 not included so continuity correction is $25 + 0.5 = 25.5$

$= P(Z > 0.8528\ldots)$

$= 1 - 0.8023$

$= 0.1977$

Look up 0.85 in table.

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2 Review Exercise

Exercise A, Question 19

Question:

a State the condition under which the normal distribution may be used as an approximation to the Poisson distribution.

b Explain why a continuity correction must be incorporated when using the normal distribution as an approximation to the Poisson distribution.

A company has yachts that can only be hired for a week at a time. All hiring starts on a Saturday. During the winter the mean number of yachts hired per week is 5.

c Calculate the probability that fewer than 3 yachts are hired on a particular Saturday in winter.

During the summer the mean number of yachts hired per week increases to 25. The company has only 30 yachts for hire.

d Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in summer.

In the summer there are 16 Saturdays on which a yacht can be hired.

e Estimate the number of Saturdays in the summer that the company will not be able to meet the demand for yachts. ***E***

Solution:

a $\lambda > 10$ or large

b The Poisson distribution is *discrete* and the normal distribution is *continuous*.

c Let Y be the random variable 'the number of yachts hired in winter'.

$Y \sim \text{Po}(5), \text{P}(Y < 3) = \text{P}(Y \le 2)$ — 'Yachts hired per week is 5'.

$= 0.1247$ — Use table.

d Y is approximately N(25, 25) — Mean number of yachts hired per week is 25.

$\text{P}(X > 30) = \text{P}\left(Z > \frac{30.5 - 25}{5}\right)$ — '30 yachts for hire' so 'cannot be met' is *greater than* 30. 30 not included, so use continuity correction $30 + 0.5 = 30.5$.

$= \text{P}(Z > 1.1)$

$= 1 - 0.8643$

$= 0.1357$

e

Number of weeks $= 0.1357 \times 16$

$= 2.17$

So 2 (or 3) Saturdays.

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2 Review Exercise
Exercise A, Question 20

Question:

The continuous random variable X is uniformly distributed over the interval $\alpha < x < \beta$.

a Write down the probability density function of X, for all x.

b Given that $E(X) = 2$ and $P(X < 3) = \frac{5}{8}$, find the value of α and the value of β.

A gardener has wire cutters and a piece of wire 150 cm long which has a ring attached at one end. The gardener cuts the wire, at a randomly chosen point, into 2 pieces. The length, in cm, of the piece of wire with the ring on it is represented by the random variable X. Find

c $E(X)$,

d the standard deviation of X,

e the probability that the shorter piece of wire is at most 30 cm long. ***E***

Solution:

a $$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha < x < \beta \\ 0, & \text{otherwise} \end{cases}$$

b $E(X) = 2, \quad P(X < 3) = \frac{5}{8}$

i.e. $\dfrac{\alpha + \beta}{2} = 2 \quad \dfrac{3 - \alpha}{\beta - \alpha} = \dfrac{5}{8}$

Use properties of uniform distribution to form 2 equations.

$$\begin{aligned} \alpha + \beta &= 4 \\ 3\alpha + 5\beta &= 24 \\ \beta &= 6 \\ \alpha &= -2 \end{aligned}$$

Solve simultaneously.

c $E(X) = \dfrac{150 + 0}{2} = 75 \text{ cm}$

Use formula.

d $$\begin{aligned} \text{Standard deviation} &= \sqrt{\frac{(150-0)^2}{12}} \\ &= 43.3 \text{ (3 s.f.)} \end{aligned}$$

Use formula.

e $P(X \le 30) + P(X \ge 120)$

There are 2 ends!

$$= \frac{30}{150} + \frac{30}{150}$$
$$= \frac{60}{150} = \frac{2}{5}$$

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2 Review Exercise
Exercise A, Question 21

Question:

Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

a Test, at the 5% significance level, whether or not the proportion p of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

The manager of the supermarket thinks that the probability of a person buying a gigantic chocolate bar is only 0.02. To test whether this hypothesis is true the manager decides to take a random sample of 200 people who bought chocolate bars.

b Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.02. The probability of each tail should be as close to 2.5% as possible.

c Write down the significance level of this test. ***E***

Solution:

a $H_0: p = 0.2, \quad H_1: p < 0.2$ ← 'Lower' so one-tailed test.

Let X be the random variable 'number of people buying a family size bar'

$X \sim B(30, 0.2)$ ← Sample of 30 people; $n = 30$; 20% of people : $p = 0.2$

$P(X \leq 2) = 0.0442 < 0.05$ ← Table value.

Sufficient evidence to reject H_0.

Conclude that the proportion of family size sold is lower than usual ← Conclusion in context.

b $H_0: p = 0.02, H_1: p \neq 0.02$ ← 'is different from' so two-tailed test.

Let Y be the random variable 'number of gigantic bars sold'

$Y \sim B(200, 0.02)$ which is $Po(4)$ approximately ← $np = 200 \times 0.02 = 4$

$P(Y = 0) = 0.0183$ is closest to 0.025

$P(Y \geq 9) = 1 - 0.9786 = 0.0214$ is closest to 0.025 ← From table values look for 'as close to 2.5% as possible'

Critical region $Y = 0$ and $Y \geq 9$

c

$$\text{Significance level} = 0.0183 + 0.0214 = 0.0397$$

i.e. 3.97%